Opening (20 min)

Open the PowerPoint file and display the mirror problem.

- We all look in the mirror, probably several times a day. Have you ever looked in the mirror and thought about why you see an image, where is the image, and how big is the image?
- Imagine you are standing in front of a mirror, and you make two marks on the mirror…one at the top of your head and one at your chin. (Advance PPT)
- You measure the distance between these two marks. How does this distance compare with the height of your head? Are they the same? Is the distance that you marked bigger or smaller than the height of your head? (Advance PPT)
- Does your distance from the mirror have anything to do with the size of the image? (Advance PPT)
- How large would the mirror need to be for you to see your entire body from head to toe?

Let's explore this concept with a virtual manipulative in GeoGebra. (Open the mirror point file)

- The moveable blue point represents an image in front of the mirror. There are rays of light coming from the object.
- (Check Reflection box) We can see the path that the light rays would take as they come from the object and bounce off of the surface of the mirror.
- (Check All Rays box) Here are the paths of all three rays, and the angles or reflection that result.
- (Check Eye box) When we look in the mirror, we see an image. We'll call that our virtual image. Where does this virtual image appear to be? Behind the mirror? So it may help to see the virtual rays that create the image that we see.

Now let's go back to our original investigation. (Close the mirror point file and open the mirror image file)

- In this example, we have a young girl standing in front of a mirror. She looks at her foot in the mirror. (Check Foot Ray box)
- We can measure the angles of incidence and reflection, and see that as we move the girl closer to the mirror, and then further away, these angles remain congruent. (Move the girl close to the mirror and then away by using the yellow point)
- One of the marks we made was at the top of our head. (Check Head Ray box) We could measure those angles as well, and we would find that they are congruent.

- In the last example, we looked at the virtual rays that create the image that appears “behind” the mirror. So here they are… (Check Virtual Ray box)
- (Check Virtual Image box) …and here is the image that we see, that appears to be behind the mirror, the virtual image.

Let’s try and answer the first question that was posed.

- Look at the distance from the point on top of the young girl’s head, to the point on her eye. (Check Vertical + Horizontal box) Does this distance remain the same in the virtual image?
- What about the surface of the mirror, where we made the marks? If we were to make a mark on the mirror directly at her eye level, and then another on the mirror’s surface where the top of her head appears to be…How does this distance compare to the actual distance between the top of her head and her eye? (Move the girl closer, and then further away) Any conjectures?
- Let’s look at the image on the mirror’s surface. (Check Image on Mirror) (Trace (with an arrow or laser pointer) the ray from her eye, to the top of her head on the virtual image, then trace back to where that point is on the mirror’s surface. Do the same with the ray from her eye to her virtual foot image, then back to that point on the mirror’s surface) So here is the top of her head on the mirror’s surface, and here is her foot. The dark segment is where the young girl sees her entire body on the surface of the mirror. (Do not move the girl yet)
- Going back to one of our original questions… How large would the mirror need to be for you to see your entire body from head to toe? Does your distance from the mirror have anything to do with it? Well, here we see that the young girl can see her entire body using only THIS much of the mirror. What is she moves closer? Will she need to use more of the mirror’s surface to see her entire body? What if she moves away? Would she be able to use a smaller mirror, and still see herself from head to toe? Any conjectures?
- Well, let’s check it out. (Move the girl slowly towards the mirror and point out the image on the surface of the mirror) Anything unexpected? What if she moves back? (Move the girl slowly away from the mirror). Hmmm. What is our observation about the image on the mirror’s surface? Is it changing at all? No. Why is that? It isn’t what we might expect.
- Let’s see if we can prove why this is happening. What are some of the geometric properties at play here? (Move the girl closer and then away a few times) Observations? Angles, parallel lines, triangles, congruent distances? (Point out the similar triangles and the midpoints of the sides of the large triangle, lead participants to the discovery of the midsegment, and that the image on the mirror will always be half length of the actual image)

Installation of GeoGebra and Freeplay (10 min)

Instructions for downloading the software were sent to you before the workshop, but here there are again.

Installation WITH Internet access

- Go to http://www.geogebra.org
  Home page will look like this:

- On the left side you will find the tab "Download". Click on download

- You will see this window:

It is recommended to use GeoGebra WebStart which guarantees that you are always running the latest version of GeoGebra, eliminating complicated installation or upgrade procedures. Note: you can use GeoGebra WebStart offline too.

Click on "GeoGebra WebStart". This new window will appear:

Select "GeoGebra WebStart". Follow the steps indicated in the windows that come up in the screen. Confirm all messages that might appear with ***OK*** or ***YES.***

Installation WITHOUT Internet access

- Distribute installer files via USB drives or CDs
- Copy installation file from storage device into the created GeoGebra_Workshop folder on the computer. There are four to choose from depending on your computer system.
- Double-click the GeoGebra Installer file and follow the instructions of the installer assistant.

Introduction to GeoGebra (10 min)

Background Information About GeoGebra

GeoGebra's User Interface

- Graphics window
- Geometry tools
- Toolbar help
- Show / hide coordinate axes and grid
- Algebra window
- Input field
- Show / hide algebra window and input field
- Menubar

Basic Use of Tools

- Activate a tool by clicking on the button showing the corresponding icon
- Toolboxes contain similar tools or tools that generate the same type of new object.
  Open a toolbox by clicking on the lower part of a button and select another tool from this toolbox. You don't have to open the toolbox every time you want to select a tool. If the icon of the desired tool is already shown on the button it can be activated directly.
- Check the toolbar help in order to find out which tool is currently activated and how to operate it.
- Selecting an already existing object: When the pointer hovers above an object, the pointer changes its shape from a cross to an arrow and the object highlights. Clicking selects the corresponding object.
- To correct mistakes step-by-step use the *Undo* and *Redo* buttons located in the upper right hand corner or find the commands under the *Edit* menu.

Constructing Rectangles – An Initial Exploration (20 min)

Presenter Information

- *Objectives*
  - *Learn how to use the various tools such as, New Point, Perpendicular lines, Parallel lines, intersection point, and Polygon, Line through 2 points.*
  - *Investigate how to create a rectangle that stays a rectangle.*

Introduction of tools

In this section the following tools need to be (re)-introduced. Please make sure participants try them out and get comfortable using them.

Introduction of features

Additionally, participants should also know how to use the following features:

	New Point	Place points that are both free to move and points that are on specific objects
	Parallel lines	Point line runs through, line to be parallel to
	Intersection of two objects	2 objects that intersect
	Polygon	Click the vertices of the polygon
	Line through two points	Click the two points line will run through
	Perpendicular line	Creating a perpendicular line Required objects: straight object and point
	Show / hide object	Showing / hiding objects Note: Activate another tool to apply changes

- How to navigate tool bars
- Move tool
- Show / hide algebra window (*View* menu)
- Show / hide axes (*View* menu)
- Show / hide grid (*View* menu)

Challenge: Creating a rectangle

- *Overview*
  - *Participants will create a rectangle not locked to the axes.*
  - *Participants may be recommended to use the tools below*
- *Note: You may begin the participants with a line through 2 points but otherwise let them work on their own.*

Presenter is to begin by telling the participants to construct a rectangle. The presenter may give the list of tools below as possible tools to use in order to complete this task but allow participants time to work and figure it out on their own.

	New Point	Place points that are both free to move and points that are on specific objects
	Line through two points	Click the two points line will run through
	Perpendicular line	Creating a perpendicular line Required objects: straight object and point
	Parallel lines	Point line runs through, line to be parallel to
	Intersection of two objects	2 objects that intersect
	Polygon	Click the vertices of the polygon
	Show / hide object	Showing / hiding objects Note: Activate another tool to apply changes

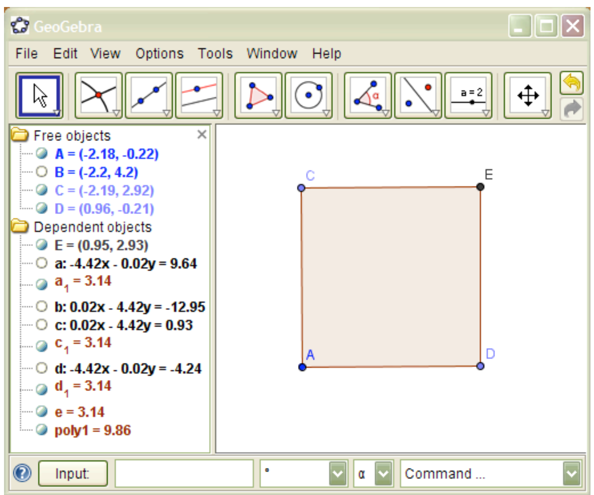
Questions to ask:

1) If you move the points on your rectangle does it stay a rectangle? If not why? (ask how they constructed it)
2) What is the difference between a true construction and just building a rectangle? (mention to participants that anything that can be constructed with a straight edge and a compass can be constructed in GeoGebra)
3) What are the properties of a rectangle?

Guided Practice: Creating a rectangle

- *Overview*
  - *Participants create multiple types of points: points locked to the lines, intersection points and free moving points.*
  - *Create perpendicular lines (also show Parallel lines)*
  - *Participants will create a rectangle that can change size*
- *Note: make sure everyone locks points correctly before creating polygon*

To transition from the discussion in the challenge to this activity show the participants a rectangle that is constructed correctly as shown below.



Guided construction

Preparations

- Open a new GeoGebra file
- Hide the axis /grid/algebra window
- Show new points only

Construction steps

1		Create point A (Discuss that point A is free moving and discuss algebra window)
2		Create Point B
3		Create a line through points A and B (Discuss that you did not need the two points there to begin with and how the line can be moved. Also discuss the labeling how GeoGebra names items)
4		Hide point B (Discuss how it only highlights the point until you change tools and you can right click on an object to do the same thing)
5		Create point C on line a (discuss the color change and how the point moves)
6		Create a line perpendicular to line a and through point A, line b (move it around and show it stays perpendicular)
7		Create a line perpendicular to line a and through point C, line c
8		Place point D on one of the new lines b or c
9		Create a line parallel to line a and through point D, line d
10		Create a point at the intersection created with line d that is not yet marked, point E (Discuss how this point is grey and does not move on its own. Also discuss how you can miss the intersection when you click)
11		Create a polygon using points A, C, D, E (Discuss that if they miss a point a new point will be created and they must undo. Also they must click the first point again in the end to close the polygon.)
12		Hide the lines that are not need

* During the construction have a discussion about why they must move objects as they go don’t just assume it is right and check at the end.

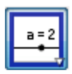
Guided Practice – Triangle Inequality Theorem (60 min)

Preparation

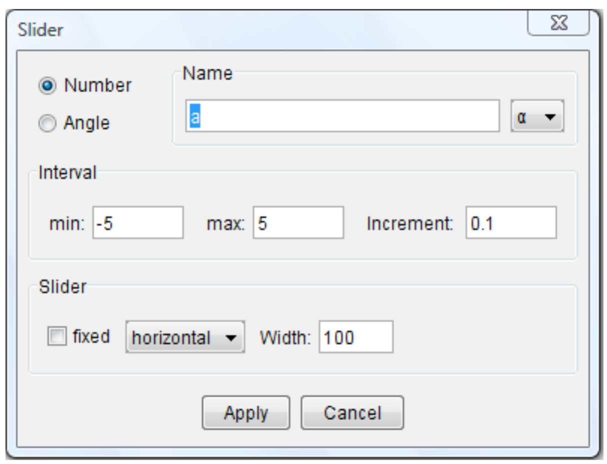
- Open a new GeoGebra file
- For this construction, we will not use the coordinate axes or the Algebra window. Click on the *View* menu on the top of the page. Select *Axes* and click on it to inactivate it. Open the *View* menu again, select *Algebra* window and click to hide it or click on the *x* located on the upper right hand side of the algebra window.

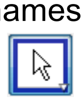
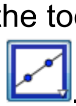
Construction process

We are going to construct a triangle with given side lengths and later be able to change the side lengths. For that we are going to create sliders.

- Select the slider tool 

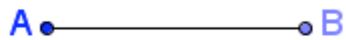
 and click on the drawing pad. A new window will appear.



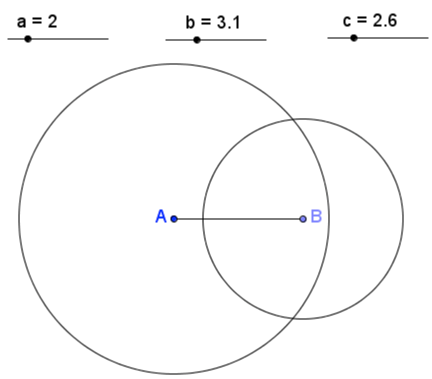
- The slider will represent the length of one side of the triangle. The name of the Number (slider) is by default *a*. The default settings suggest a minimum value of -5 and maximum value of 5. We are going to change the minimum value to 0 and the maximum to 10. Leave the increment as it is. Click the button *Apply*.
- We need 2 more sliders for the other two sides of the triangle. Repeat the process twice. The names of the new numbers will be, by default, *b* and *c*.
- Select the move tool  and drag the point on the sliders. Notice the numbers changing from 0 to 10 in increments of 0.1.
- Select the tool *Segment with given length from point,* found under the *Line* toolbox . Click anywhere in the drawing pad to create a point. A window will

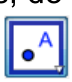
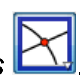
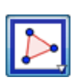
appear asking for the length. Type the letter *c* (name of the slider). The segment will be created. Select the move tool and change slider c. Notice how the segment increases and decreases in length as the value in the slider changes.

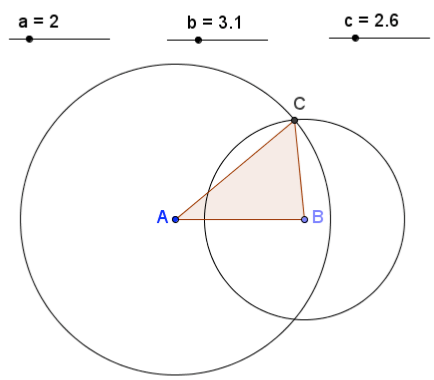
- Right-click (MacOS: Ctrl-click) on one of the endpoints of the segment, select *show label*. Do the same for the other end point.



- In order to construct the other two sides of the triangle, we will use the *Circle with center and radius* tool , located in the circle toolbox. Click on point A (center of the circle). A window will appear asking for the radius, type letter *b*. A circle will be constructed that has radius *b*. Select the *Move* tool and change slider *b*. Notice how the radius of the circle changes.
- Activate the *Circle with center and radius* tool again. Click on point B (center of the circle and this time type letter *a* for the radius of this last circle. Move the sliders, verify your circles change as the values of the sliders *a* and *b* change.



- If your two circles, do not intersect, move slider *a* or *b* until they intersect. Under the *point* toolbox  select *Intersect two objects* . Move the pointer close to the intersection, notice the two circles getting darker, click on the intersection to create the point.
- Right-click (MacOS: Ctrl-click) on the intersection point and select *Show label*. This will be point C.
- Using the Polygon tool  click on the points A, B, C, and A in order to create the vertices of a polygon. Connect the last and first vertex to close the polygon. Always connect vertices counterclockwise! (this is necessary for measuring the inside angles of the polygon)



- Select the move tool and move the points on the sliders. Notice how the triangle changes. Do we always have a triangle? Make a conjecture about your observations, what length should the sides be so we can have a triangle?

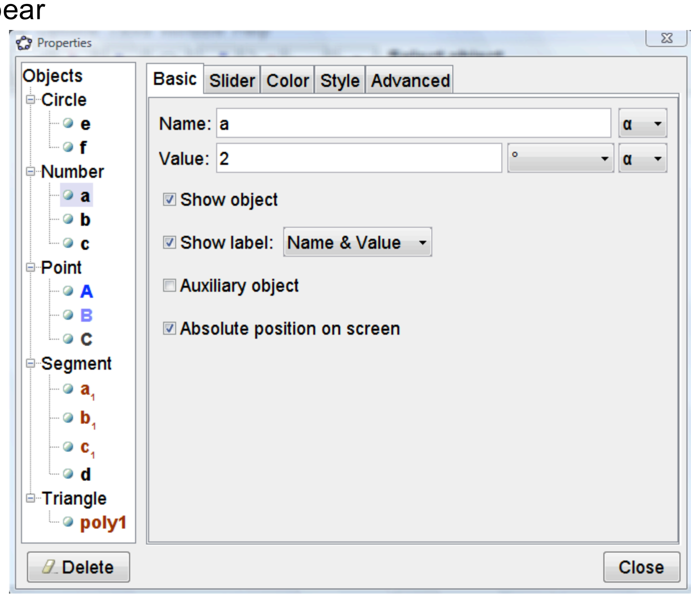
Use the Properties dialog to enhance your construction

There are different ways to access the Properties dialog:

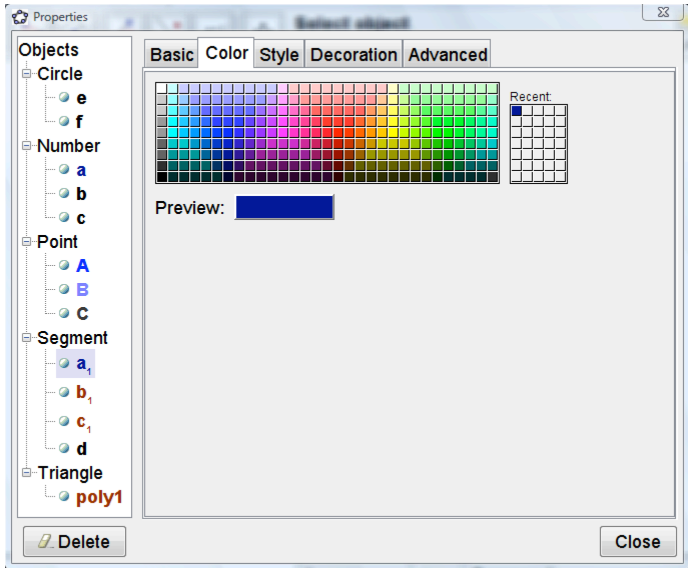
- Right-click (MacOS: Ctrl-click) on an object. Select *Properties…* from the Edit menu
- Double click an object in *Move* mode

We will change the colors of the sliders, the sides of the triangles, and the circles so we can visually tell what slider changes what side.

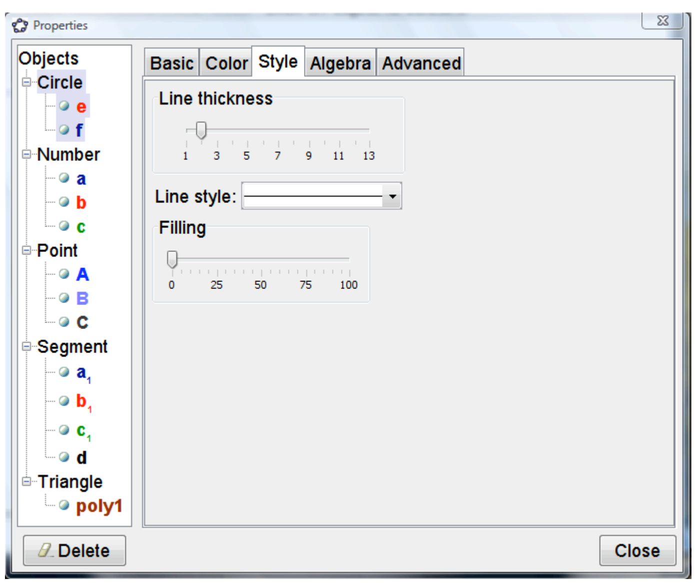
- Right-click (MacOS: Ctrl-click) or double click on slider *a* and select *Properties*. A window will appear



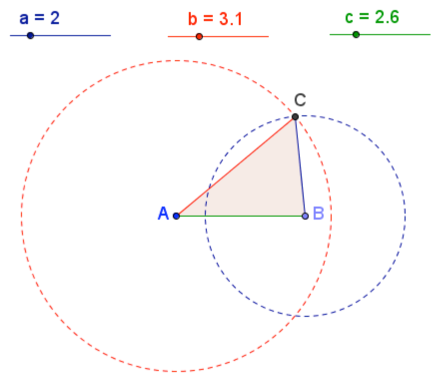
- Select the tab Color and pick any color you like



- From the list on the left side of the Properties window, find Segment: a_1. Select segment a_1 by clicking on the name a_1 and use the same color you selected for number *a*. Select circle f and use the same color.
- On the left list, select number *b* and pick a different color. Use the same color for segment b_1.
- Finally, select number *c* and a third color. Use the same color for segment c_1 and circle *e*.



- On the left, click on the work *Circle* to select both circles at the same time, under the tab *Style* you will find *Line style*, click on the down arrow and select a dashed style for the circles.



  - Close the Property window.

Measuring the length of the sides of the triangle

On the *Measurement* tools menu, , find the *Distance or length tool*, . This tool could be used in two different ways:

a. To measure distance between 2 points
b. To measure length of a segment

Use the tool, either by clicking on 2 endpoints at a time or by clicking on a segment. Find the length of the 3 sides of the triangle; verify that is the same as the values on the sliders.

Inserting text into the Drawing pad

Text can be static and dynamic.

Inserting static text

Activate the *Text* tool and click anywhere on the drawing pad.

- Type the following text into the appearing window:
  "*Triangle Inequality Theorem:*
  The sum of the lengths of any two sides of a triangle is greater than the length of the third side."
- Click *Apply*.

- Adjust the position of the text using the *Move* tool.

Hints: You can change the properties of the text in the *Properties dialog* (e.g. edit text, font style, font size, formatting). On tab *Basic* you can fix the position of the text so it can't be moved accidentally any more.

Inserting dynamic text

Dynamic text refers to existing objects and adapts automatically to modifications, for example the length of the sides as the sliders move. In our example, we want to show that the triangle will exist when the following conditions are true: a + b ≥ c, a + c ≥ b, and b + c ≥ a

- Activate the *Text* tool and click on the drawing pad.
- Type a + b ≥ c into the appearing window, and click *Apply*.

  Hints: to find the ≥ symbol, open the window located on the right hand side of the text dialog box.
  This is static text and won't change if the sliders are moved.

- With the *text* tool still activated, click on the drawing pad again. Insert dynamic text by clicking on segment *a* in the graphics window.
  - GeoGebra will insert the name of the segment into the text field
- To add the + sign, type the following: (do not leave an empty space)+" + ". Only leave spaces inside the quotation marks.
  - The first + sign connects the dynamic and static part of the text
  - Quotation marks around any text indicate it will be static.
- Click on segment *b.*
  - Additionally, GeoGebra adds a + symbol to connect the new dynamic part to the rest of the text.
- Add the = sign by typing +" = "
- Click on segment *c.*
- Click *Apply*.
- Observe how the dynamic text changes as you move the sliders

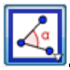
Task:

Insert the static and dynamic text for a + c ≥ b and b + c ≥ a.

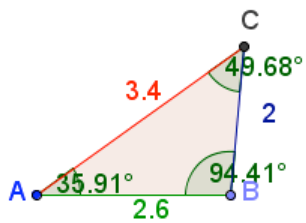
Hide/Show Objects

The circles were necessary for the construction of the triangle but, for the following investigations, are not necessary. Activate the *Hide/Show Objects* tool and click of the circles. You will still see them while the tool is active. Activate the *Move* tool, the circles will disappear. If you delete the circles, the triangle will disappear because its construction is dependant on those circles.

Interior Angles

We can find the measures of the interior angles of the triangle by using the *Angle* tool .

- Activate the tool
- Click one time inside of the triangle. GeoGebra will give you the measure of the three interior angles at once.

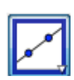
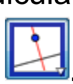


- Is there any relationship between the measure of the angle and the length of the opposite side? Make a conjecture. Move the sliders to verify your conjecture.

Area of the triangle and Altitudes

- Hide the interior angles
- Activate the *Area* tool  and click inside of the triangle
- You can move the text by activating the *Move* tool and dragging the text.

If we assume side AB to be the base of the triangle and knowing the area, what is the altitude?

- Using the *Line trough two points* tool  draw line AB by clicking on point A and then point B.
- The altitude is perpendicular to side AB and passes trough point C. Activate the *Perpendicular line* tool . Click on point C and then on line AB (you can read the instruction in the help menu)
- Using the *Intersect two objects* tool, make a point at the intersection of the perpendicular line and line AB. You do not need to be at the intersection point, click anywhere on the perpendicular line and then anywhere on the line AB.
- Hide the perpendicular line and line AB

- Create a segment using the *Segment between two points* tool click on point C and the newly created point of intersection.
- Right-click (MacOS: Ctrl-click) on the segment and using the *Properties…* dialog, change the color, thickness or style of the segment.
- Show the right angle, use the Angle tool and click on three points counterclockwise: B, intersection, C. Right-click on the angle and click on *Show label.* This will actually hide the label since it was showing before.
- Move slider *a*. Why does the right angle change? GeoGebra measures the angles counterclockwise, if the point of intersection goes to the right of point B, GeoGebra gives you the measure of 270°.
- Find the length of the new segment. Is the length what you had calculated before? If you move slider *a* only, what is the highest value the altitude can take?
- As you move slider a, point C moves. What is the path that point C describes? Right-click on point C, select *Trace on.* Move slider *a*, is the path what you had predicted?

Saving GeoGebra files

You might want to create a new folder called GeoGebra_Workshop on your desktop to store all your work.

Save your drawing

- Open the *File* menu and select *Save*.
- Select the folder *GeoGebra_Workshop* in the appearing dialog window.
- Type in a *name* for your GeoGebra file.
- Click *Save* in order to finish this process.

Hint: A file with the extension '*.ggb*' is created. This extension identifies GeoGebra files and indicates that they can only be opened with GeoGebra.

Hint: Name your files properly: Avoid using spaces or special symbols in a file name since they can cause unnecessary problems when transferred to other computers. Instead you can use underscores or upper case letters within the file name (e.g. Triangle_Inequality.ggb).

What to practice

- How to open a new GeoGebra window (menu *File* – *New window*).
- How to open a blank GeoGebra interface within the same window (menu *File* – *New*)
  Hint: If you didn't save the existing construction yet GeoGebra will ask you to do so before the blank screen is opened.
- How to open an already existing GeoGebra file (menu *File* – *Open*).
  - Navigate through the folder structure in the appearing window.
  - Select a GeoGebra file (extension '*.ggb*').
  - Click *Open*.

Practice Block

Choice#1: Explore reflections about lines and points, translations, and rotations

Reflections (Mirror Lines)

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Reflection of a polygon with respect to a line:

1		Construct Triangle ABC
2		Construct Line DE
3		Reflect Triangle ABC with respect to Line DE

Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point A.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in opposite order. For example, the reflection of Quad ABCD could be named Quad D'C'B'A'.

Can't move Triangle A'B'C' because it is just the image of Triangle ABC. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is double the distance from any of the points to the line of reflection. In other words the line of reflection would be the perpendicular bisector of the segment joining a point and its image.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of reflecting an object with respect to a line is an isometry. A transformation that preserves distance and angle measure is an isometry.

Add a Free Point G to the worksheet. Without using the reflection tool, how would you locate point G' (reflected about Line DE)?

- Reflected Point Construction One
- Reflected Point Construction Two

If you suspected that two figures were congruent and related by a reflection, how could you confirm your guess and reconstruct the reflection line?

Reflections (Mirror Point)

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Reflection of a polygon with respect to a point:

1		Construct Triangle ABC
2		Construct Point D
3		Reflect Triangle ABC with respect to Point D

Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- Is there another transformation that is equivalent to this one?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point A.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad ABCD could be named Quad A'B'C'D'.

Can't move Triangle A'B'C' because it is just the image of Triangle ABC. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The other transformation that would be the same is a 180 degree rotation (clockwise or counter-clockwise) with respect to the same point.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of reflecting an object with respect to a point is an isometry. A transformation that preserves distance and angle measure is an isometry.

Translations

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Translation of a polygon with respect to a vector:

1		Construct Triangle ABC
2		Construct Vector DE
3		Translate Triangle ABC using Vector DE

Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point A.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad ABCD could be named Quad A'B'C'D'.

Can't move Triangle A'B'C' because it is just the image of Triangle ABC. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is equal to the magnitude of the vector. In addition, the angle at which the pre-image has been slid to the same as the relative angle of the vector..

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of translating an object with respect to a vector is an isometry. A transformation that preserves distance and angle measure is an isometry.

Rotations

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Rotation of a polygon with respect to a point:

1		Construct Triangle ABC
2		Construct Point D
3		Make angle slider
4		Rotate Triangle ABC about point D using the angle slider

Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point A.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad ABCD could be named Quad A'B'C'D'.

Can't move Triangle A'B'C' because it is just the image of Triangle ABC. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is a complicated issue in this case. Some triangle trigonometry may have to be used. Unless distance is meant to be circular distance using the radius and angle of rotation.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of translating an object with respect to a vector is an isometry. A transformation that preserves distance and angle measure is an isometry.

<u>Choice #2:</u> Explore reflecting over parallel and intersecting lines

Reflecting over parallel lines

- *Overview*
    - *Discuss how multiple reflections affect an object*
    - *Guided construction polygon reflected over two parallel lines.*

Discussion:

- How would reflecting over two parallel lines affect an object?
- Could there be other transformations that are isomorphic to reflecting over two parallel lines? If so, find one.

Guided construction polygon reflected over two parallel lines.

- Preparations
    - Open new GeoGebra file
    - Hide algebra window
    - HIde coordinate axes
    - Hide grid (if necessary)

- Construction steps

1		Construct Line AB
2		Construct Point C, not on Line AB
3		Construct a line parallel to Line AB thru Point C
4		Construct polygon (exterior of parallel lines)
5		Reflect polygon with respect to Line AB
6		Reflect reflected polygon with respect to line thru Point C

- Check construction
    - Drag the original polygon (Which is the original polygon? How do you know which is the original polygon?)

- Discussion

1. How does moving the original polygon affect the first reflected polygon?

2. How does moving the original polygon affect the second reflected polygon?

   a) Is there a single transformation that can be done on the original polygon to create the second reflected polygon?

   b) If you answer to part a) is yes, then create the transformation in GeoGebra.

   c) Can you argue why there is/isn't a single transformation that could replace the double reflection above?

Reflecting over two intersecting lines

- *Overview*
    - *Discuss how multiple reflections affect an object*
    - *Guided construction polygon reflected over two intersecting lines.*

Discussion:

- How would reflecting over two intersecting lines affect an object?
- Could there be other transformations that are isomorphic to reflecting over two intersecting lines? If so, find one.

Guided construction polygon reflected over two parallel lines.

- Preparations
    - Open new GeoGebra file
    - Hide algebra window
    - HIde coordinate axes
    - Hide grid (if necessary)

- Construction steps

1		Construct Line AB
2		Construct Line AC
3		Construct polygon (exterior of parallel lines)
4		Reflect polygon with respect to Line AB
5		Reflect reflected polygon with respect to Line AC

- Check construction
    - Drag the original polygon (Which is the original polygon? How do you know which is the original polygon?)

- Discussion

1. How does moving the original polygon affect the first reflected polygon?

2. How does moving the original polygon affect the second reflected polygon?

a) Is there a single transformation that can be done on the original polygon to create the second reflected polygon?

b) If you answer to part a) is yes, then create the transformation in GeoGebra.

c) Can you argue why there is/isn't a single transformation that could replace the double reflection above?

Guided Practice - Exploring symmetry and transformations with images

Presenter Information

- *Objectives*
  - *Learn how to use the various transformation tools such as, reflections, rotations, translations, and dilations.*
  - *Investigate how certain transformations change images in the coordinate plane.*

Introduction of tools

In this section the following tools need to be (re)-introduced. Please make sure participants try them out and get comfortable using them.

	Mirror object at line	Object to mirror, then line of reflection
	Mirror object at point	Object to mirror, then center point
	Rotate object around point by angle	Object to rotate, then center point and angle
	Translate object by vector	Object to translate, then vector
	Dilate object from point by factor	Object to dilate, then center point and factor
a = 2	**Slider**	Click on drawing pad to specify position of slider

Introduction of features

Additionally, participants should also know how to use the following features:

- Navigation bar to review construction steps (*View* menu)
- Construction protocol (*View* menu)
- Show / hide algebra window (*View* menu)
- Show / hide input field (*View* menu)
- Show / hide grid (*View* menu)
- Show / hide axes (*View* menu)
- Input equations (*Input Field*)
- Enter text, dynamic (*Text* tool)

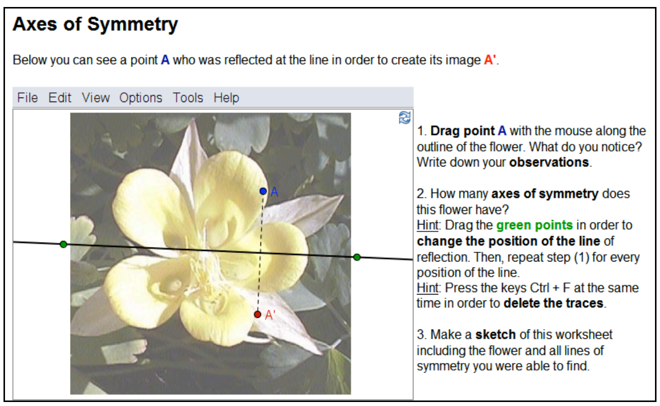
Activity 1: Drawing Tool for Symmetric Figures

- *Overview*
  - *Participants experience using a dynamic worksheet*
  - *Create a drawing tool for symmetric figures*
  - *Insert a background picture*
  - *Participants check picture for axes of symmetry*
- *Note: Check if everybody has access to a suitable picture file!*

Open dynamic worksheet, Drawing_Tool_Symmetry.html, and show how students can use it to explore the axes of symmetry of an inserted picture.

Motivate participants by telling them that they will be able to create such a dynamic worksheet at the end of this workshop day!

Discussion: How could students benefit from this prepared construction?



Discuss which tools were used in order to create the dynamic figure. <u>Hint</u>: Let participants guess first!

	New point		Mirror object at line **New!**
	Show / hide label **New!**		Move
	Line through two points		Insert image **New!**
	Segment between two points		

Introduction of new tools:

Explain and demonstrate the use of new tools

- *Mirror object at line* tool
  Hint: Line or reflection needs to be created first! Point to mirror can be created on the fly.
  Hint: Order of clicks is important: First select the line of reflection, then the object to be mirrored!
- *Insert image*
  Hints: Participants need to know where they saved the provided picture file – let them search before introducing this tool!

Remind participants…

- …to read the toolbar help if they don't know how to use a tool!
- …that for some tools the order of actions / clicks is relevant!

Guided construction

Preparations

- Open a new GeoGebra file
- Hide algebra window, input field, and coordinate axes if necessary

Construction steps

1		Create a new point A
2		Show label of new point
3		Line of reflection through two points
4		Mirror point at line to get image A' Hint: Move point A to show the symmetric movement.
5		Segment between point A and its image A'
7		Feature *Trace on* for points A and A′
8		Move point A to draw a dynamic figure

9		Save the construction

Discuss characteristics of feature *Trace on*

- The trace is a temporary phenomenon
  Hint: Whenever the graphics are refreshed, the trace disappears!
- In order to delete the trace, refresh the views
  Hint: Select item from the *View* menu or press keys *Ctrl* and *F* at the same time.
- The trace can't be saved and is not shown in the algebra window

Enhancing the construction

Note: Have participants make sure they know where to find a suitable picture file on their computer!

Construction steps

1		Insert image into the drawing pad Hint: Click in the lower left corner of the graphics window.
2		Adjust the position of the inserted image
3		Set the image as *background image* Hint: Open the properties dialog using the *Edit* menu.
4		Reduce the *filling* of the image
5		Save the construction.

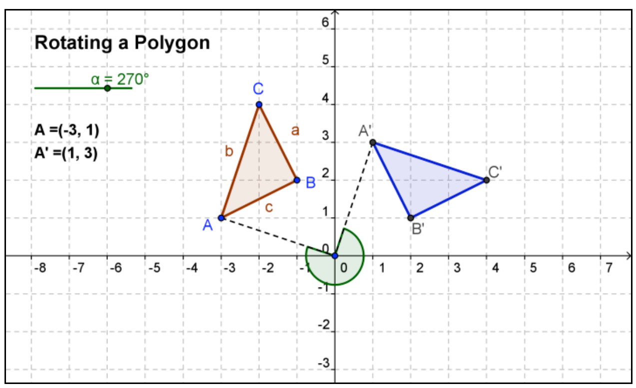
Activity 2: Rotation of a Polygon

- *Overview*
  1. *Participants experience exploring a mathematical concept as GeoGebra activity*
  2. *Demonstrate how to insert a slider*
  3. *Rotate a polygon around a point by an angle*
  4. *Insert static text into the graphics window*
  5. *Insert dynamic text into the graphics window*

Motivation task

Open the GeoGebra file, Rotation_polygon_text_slider.ggb, and have participants explore the dynamic figure on their own.

Discussion: How could teachers use this file to introduce the concept of rotation around the origin of a coordinate system to their students?



Discuss which tools were used in order to create the dynamic figure
<u>Hint</u>: Let participants guess first!

	Polygon		Segment
	New point		Angle
	Slider	ABC	Insert text
	Rotate object around point by angle		Move

Introduction of new tools:

Explain and demonstrate the use of new tools

- *Slider*
  Hint: A slider is the graphical representation of a number. Every free number can be displayed as a slider in the graphics window.
- *Rotate object around point by angle*
  Hint: This is a challenging tool which needs to be explained very thoroughly! The order of actions is relevant and several objects need to be created prior to using this tool.
- *Insert text*
  Hint: Objects need to be created prior to using their names within dynamic text.
  Hint: Creation of dynamic text without having to worry about the syntax: Type in the static part of the text (e.g. A =), then click on the object whose value you want to insert (e.g. point A). GeoGebra automatically creates the correct syntax for the dynamic text.

Remind participants…

- …to read the toolbar help if they don't know how to use a tool!
- …that for some tools the order of actions / clicks is relevant!

Guided construction

Preparations

- Open a new GeoGebra file
- Hide algebra window and input field if necessary
- Show the coordinate axes and the grid
- Open the *Properties dialog for drawing pad*:
  - On tab 'Axes' – 'xAxis' change the 'Distance' for x-Axis to 1
  - On tab 'Axes' – 'yAxis' change the 'Distance' for y-Axis to 1
  - Close the Properties dialog

Construction steps

1		Triangle ABC
2		New point D at the origin of the coordinate system
3		Rename the new point to O
4		Slider for angle α Note: Check 'Angle'; Increment 90˚

5		Rotate triangle ABC around point O by angle α Hint: Counter clockwise rotation
7		Segments AO and A'O
8		Angle AOA' Hint: Hide the label of this angle.
9		Move slider and check the image of the triangle
10		Save the construction

Enhancing the construction

Construction steps

1	ABC	Insert static text: ***Rotation of a Polygon***
2	ABC	Insert dynamic text: ***"A = " + A*** Hint: Enter *A* = without the quotation marks, then click on point A in the graphics window. GeoGebra automatically inserts the missing quotation marks and the + sign.
3	ABC	Insert dynamic text: ***"A' = " + A'***
4		Move slider and text to desired positions
5		Fix slider so it can't be dragged around any more Hint: Properties dialog – tab 'Slider' – check 'fixed'
7		Fix text so it can't be dragged around any more Hint: Properties dialog – tab 'Basic' – check 'Fix object'
8		Save the construction

Activity 3: Reflecting and Distorting a Picture

In this activity you are going to use the following tools. Please, make sure you know how to use every single tool before you begin. Also, check if you have the picture Sunset_palmtrees.png saved on your computer.

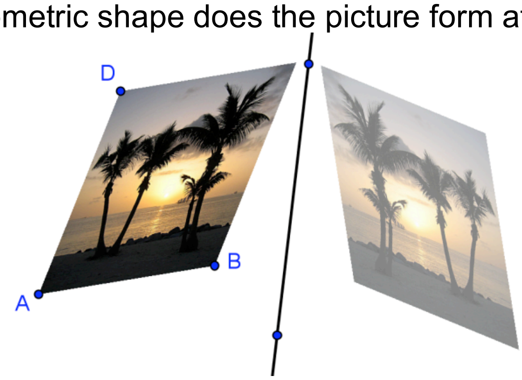
	Insert picture		Parallel line
	Move		Intersect two objects
	New point		Segment between two points
	Line through two points		Angle
	Mirror object at line		

Construction process: Reflecting a picture

1. Close the algebra window and hide the coordinate axes.
2. Insert picture Sunset_palmtrees.png into the left part of the graphics window.
3. Create a new point A at the lower left corner of the picture.
4. Set point A as the first corner point of your picture.
   Hint: Open the *Properties dialog* and select the picture in the list of objects. Click on tab 'Basic' and select point A from the drop-down list next to *Corner 1*.
   Task: How does moving point A affect the picture?
5. Create a new point B by entering B = A + (3, 0) into the input field. Set point B as the second corner point of the picture.
   Hint: You just change the size of the picture so it is 3 cm wide.
6. Create a vertical line through two points in the middle of the graphics window.
7. Mirror the picture at the line in order to get its image.
   Hint: You might want to reduce the filling of the image in order to be able to better distinguish the original from the image.
8. Task: Move the picture with the mouse and observe how this affects the image.
9. Task: Move the line of reflection by dragging the two points with the mouse. How does this affect your construction?

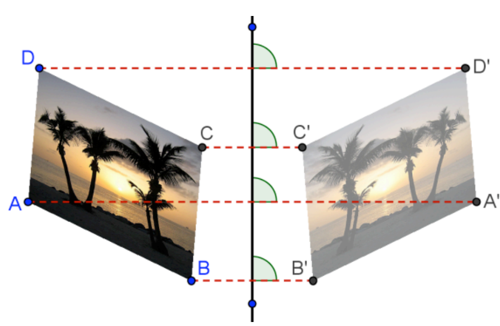
Challenge I: Distorting a picture

1. Delete the original point B in order to restore the picture's original size.
2. Create a new point B at the lower right corner of the original picture and set it as the second corner point.
   Task: How does moving point B affect the picture and its image?
3. Create a third point D at the upper left corner of the original picture and set it as the fourth corner point.
   Task: How does moving point C affect the picture and its image?
4. Task: Why doesn't it make sense to also set the third corner point of the picture?
5. Task: Which geometric shape does the picture form at any time?



Challenge II: Exploring properties of reflection

1. Construct the third corner point C of the original picture.
   Hint: Use the three corner points of the original picture, as well as parallel lines.
2. Reflect all four corner points at the line to get their images.
3. Connect corresponding points with segments (e.g. points A and A')
4. Display the angles between the line of reflection and the segments connecting corresponding points.
5. Task: Move the corner points of the original picture, as well as the line of reflection. What do you notice about the angles?



Guided Activity : Visualizing Expressions, Transformations of linear

Visualizing Expressions

- *Objectives*
  - *Use a slider to evaluate different types of expressions for multiple input values.*
  - *Visualize relations or sets of ordered pairs that are governed by such expressions using the trace point feature.*
  - *Define and discuss domain and range in the context of sliders and ordered pairs.*
  - *Graph functions using the input field.*

Introduction

In the following activities, participants will use Geogebra to explore the world of relations and functions. For a particular algebraic expression, an ordered pair will be created. The abscissa, or *x*-value, of this ordered pair will be linked to slider *a*. The ordinate, or *y*-value, of this ordered pair will be the algebraic expression in terms of slider *a*. The set of ordered pairs A = (a, 2a – 1), where the value of *a* is changing by a set increment, over a set interval, is called a **relation**. Participants will explore different relations as they restrict domain and visualize range along the way.

Preparations

In this activity you are going to use the following familiar tools. Please, make sure you know how to use each tool before you begin.

a = 2	Slider	ABC	Insert Text
	Perpendicular line		Show / hide object
	Intersect two objects		Move Drawing pad
	Segment between two points		Zoom in

	Move		Zoom out

Preparing the Window

- Open a new GeoGebra file.
- Hide the algebra window. Turn the axes on and the grid on. Set labeling to new points only.
- Use the following tools to view a window that goes from -14 to 18 on the *x*-axis and from -8 to 12 on the *y*-axis. This window does not need to be exactly these specifications.

	Move Drawing pad
	Zoom in
	Zoom out

Once the window is set, participants are ready to create Point A. Point A will be based off of the value of slider *a* and the expression $2a-1$.

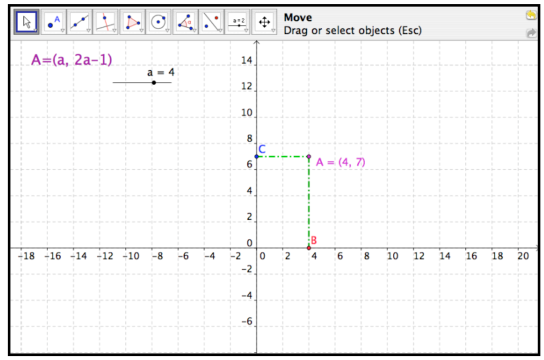
$$A = (a, 2a-1)$$

Construction Steps

1	a = 2	Create slider a. Set the minimum at -10 and the maximum at 10. Set the increment to 0.1.
2	ABC	Create a text box: A = (a,2a-1). Color the text dark purple and set the font at 18.
3		In the input, field enter the point: A=(a, 2 a-1)
4		Move slider *a* and watch Point A move through the coordinate plane.
5		Change the properties of Point A. Make it dark purple. Show its name and value.
6		Construct a line through Point A perpendicular to the x-axis. Also, construct a line through Point A perpendicular to the y-axis.
7		Construct the intersection point B, on the x-axis, and point C, on the y-axis, where the perpendicular lines created above

		intersect each axis. Color point B red and point C blue.
8		Hide the perpendicular lines.
9		Construct segments AB and AC. Make them thick, green, dashed lines.

Move slider *a* back and forth. Have the participants describe the path that point A is traveling on.



What does point B represent in relation to our expression 2*a*-1?Point B is keeping track of our input values, or domain.What does point C represent in relation to our expression 2*a*-1?Point C is a keeping track of our output values, or range, for each input value into the expression.

Have the participants trace points A, B, and C. Discuss the concept of domain and range of a relation. Have the participants zoom out so they can see the entire set of points, the entire domain and entire range. They will have to retrace it again. Change the increment for slider *a* and trace the path again. Discuss how sliders relate to domain. Have the participants consider allowing slider *a* to go from negative infinity to infinity, using all values in between. What would the graph look like? What would the domain be then? The range?

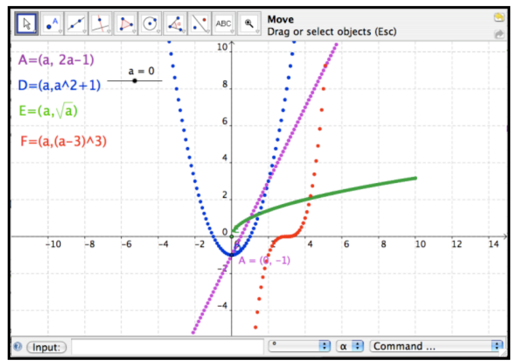
We will now add some new points into the coordinate plane. Before we do this, hide points B and C. Turn the trace off of point A. Also hide segments AB and AC. Only point A should be visible.

Construction steps

1	ABC	Create a text box: D = (a,a^2+1). Color it blue.
2	ABC	Create a text box: E = (a, \sqrt{ a }) . Use latex formula. Color it green.

3	ABC	Create a text box: F = (a, (a-3)^3) Color it red.
4		In the input, field enter the point: D = (a,a^2+1). Color it blue.
5		In the input, field enter the point: E = (a, sqrt(a)) Color it green.
6		In the input, field enter the point: F = (a, (a-3)^3) Color it red.

Move slider *a* and watch points A, D, E, and F travel throughout the coordinate plane. Trace the above points and compare their graphs. Have the participants explain their shape with respect to the operations within each expression.



Visually compare the domain and range of the graphs. Have the participants express the domain and range of each.

Why do some graphs seem to bunch up in certain spots?

To bring forth the paths on which our ordered pairs are traveling, we will enter our expressions as functions into the input field.

Construction steps

1	Into the input field, enter the formula: f(x) = 2 x - 1
2	Into the input field, enter the formula: g(x) = x^2 + 1
3	Into the input field, enter the formula: h(x) = sqrt(x)
4	Into the input field, enter the formula: j(x) = (x-3)^3

Have the participants turn off the trace feature from the moving points and watch the points move along their functions. A discussion should take place about how well Geogebra allows students to visualize expressions. From an early age, students can start associating algebraic expressions with their respective graphical representations.

Transformations of Functions

- *Objectives*
  - *Use a slider to explore the affects of certain parameters on a function.*
  - *Perform operations on functions.*
  - *Explore a dynamic worksheet to find visual solutions absolute value equations and inequalities.*

Introduction

In this section, participants will transform functions throughout the coordinate plane using sliders. The following transformations will be investigated:

$$h(x) = a \bullet f(x)$$

$$h(x) = f(x) + a$$

$$h(x) = f(x - a)$$

A pre-made Geogebra file will be used to help participants see why visualizing functions can lead to a deeper understanding of solutions to equations and inequalities.

Preparing the Window

- Open a new GeoGebra file.
- Keep the algebra window open and the axes on. Turn the grid on. Set labeling to new points only.

Construction Steps

1	a = 2	Create slider a. Set the minimum at -10 and the maximum at 10. Set the increment to 0.1.
2		Into the input field, enter the formula: f(x) = 2x-3
3		Grab the function with the mouse and move it. Watch the equation in the algebra window change with as you drag it.
4		Move it back to where it was f(x)=2x-3
5		Into the input field, enter the formula: h(x) =a*f(x). Color this function blue.

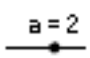
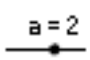
Have participants move slider *a* and discuss how and why the function behaves the way it does due to the change in *a*. When you are satisfied with the groups understanding, proceed to explore the other two transformations by redefining *h(x)*: $h(x) = f(x) + a$ and $h(x) = f(x - a)$.

Participants should bring out that $h(x) = a \bullet f(x)$ has transformed $f(x)$ by a compression towards or away from the *y*-axis depending on the value of a. If *a* is negative, as in the case of the absolute value function, the graph is reflected across the *x*-axis, and opens down.

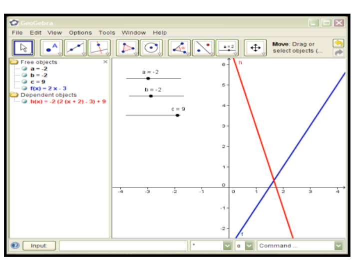
Participants should also state that $h(x) = f(x) + a$ is a vertical shift where $h(x) = f(x - a)$ is a horizontal shift. Make the participants explain why $h(x) = f(x - a)$ causes a horizontal shift *a* units to the right.

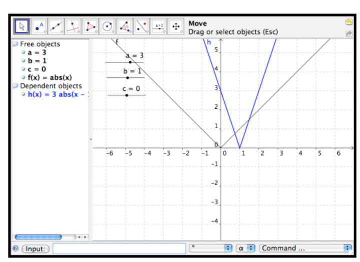
Create two new sliders, b and c, so that our function can be transformed by all three of the above transformations. Let

$$h(x) = a \bullet f(x - b) + c$$

1		Create slider b. Set the minimum at -10 and the maximum at 10. Set the increment to 1.
2		Create slider c. Set the minimum at -10 and the maximum at 10. Set the increment to 1.
3		Into the input field, enter the formula: h(x) =a*f(x - b) + c. Color this function blue.

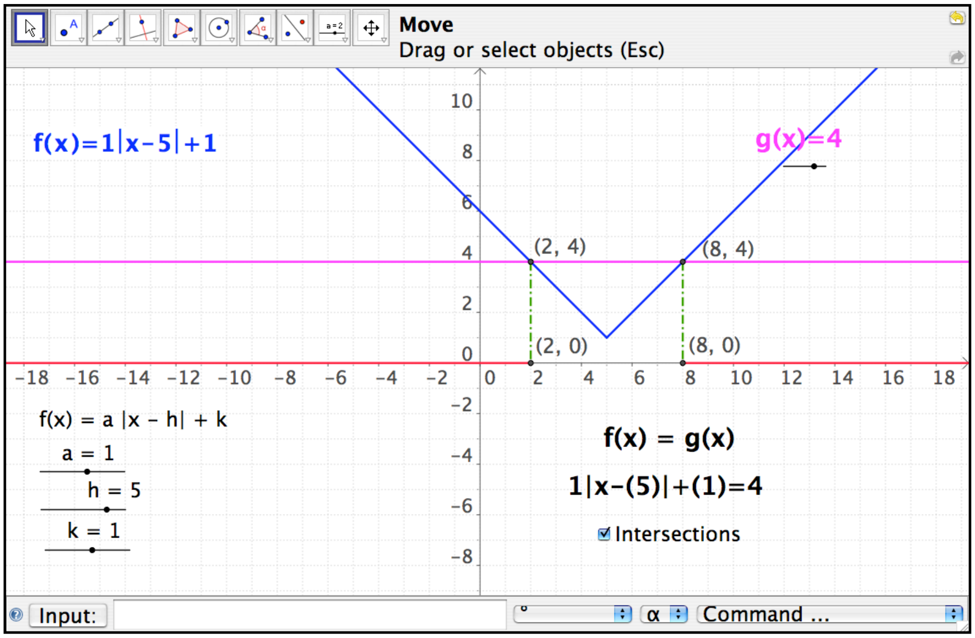
Transform the function about the plane. As an extension, redefine the original function f(x) to some other function like $f(x) = abs(x)$.





Have the participants open the GeoGebra file absolutevalue_worksheet2.ggb. Allow time for them to explore the file. Pose the following question and discuss with the group as they explain how they would use the given file.

How can visualizing functions lead to a deeper understanding of solutions to equations and inequalities?



Finding the Equation of a basketball free-throw path

If time permits

In this activity, participants will insert an image and using sliders will modify the equation of a parabola to find the equation of the path.

Guided construction

Preparations

- Open a new GeoGebra file
- Make sure the axes are showing

Construction steps

1	a = 2	Create slider a. Set the minimum at -2 and the maximum at 2. Set the increment to 0.001.
2	a = 2	Create slider b and c. Do not change the given values for the minimum, the maximum, and the increment.
3		Insert Image. (basketball_freethrow)
4		Move picture so the bottom lines up with the x-axis and the maximum point of the 60° path is on the y-axes
5		Set the image as *background image* <u>Hint</u>: Open the properties dialog using the *Edit* menu.
7		Into the input field, enter the formula: h(x) =a*(x - b)^2 + c. Change the color of this function and the thickness of the line.
8		Move sliders a, b, and c until the function overlaps the 60° path.
9		Save the construction

Question: How much would you have to translate the function so it overlaps the 45° path?

Practice Block 2: Lines

Activity 1: Slope

- *Objectives*
    - *Use a slider to explore the slope of a line.*
    - *Explore the file to answer the questions in the Exeter Lab.*

Preparing the Window

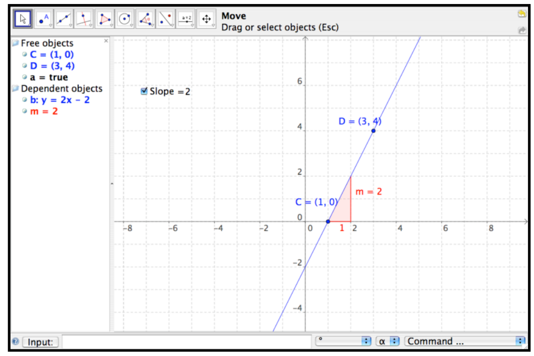
- Open a new GeoGebra file.
- Keep the algebra window open and the axes on. Turn the grid on. Set labeling to new points only.
- Set Point Capture to **on (grid)**

Construction Steps

1		Create a check box and change the caption to Slope (you will link things to the text box later)
2		Create a line with through two points (rename the points C and D and show the lable as name and value)
3		Create the slope by clicking the line created in step 2
4	ABC	Create a text box and type "=" + m (the red text will be static text)
5		Link the slope from step 3 and the text box from step 4 by highlighting them in the properties window and in the advanced tab type the name of the Boolean value (check box).

Format the file as you like lock down the text box and the sliders. Next save the file as **Linear41.ggb**. Use this file to complete the following Exeter Lab

Exeter Computer Lab - Slope



1. Once you have launched GeoGebra, click on menu **File/Open**, and choose file entitled ***Linear41.ggb***. When the file opens up, you should see a coordinate grid with a line constructed through the two points labeled C and D. The coordinates of C and D should be displayed next to the actual points. Click on the check box labeled “Slope” to view the slope. Your sketch should like the one displayed at right.

2. Using the Move tool, click on point C and drag it to the origin. Then drag point D to the location (1,1). The program is in a mode that only allows points C and D to land on points with integer coordinates. These points are known as *lattice points*, or *grid points*. Drag C and D to different locations and notice the change in the slope of the line.

   a) How would you describe lines that have positive slope?

   b) How would you describe lines that have negative slope?

   c) How would you describe lines that have a zero slope?

d) How would you describe lines that have undefined slope?

3. Drag on the line instead of either C or D and observe its behavior and the value of the slope. Summarize your observations below.

4. Move point C to the indicated location in the table below, and then find the coordinates for point D so that the slope of the line CD is the number in the left column.

Slope	**Coordinates of C**	**Coordinates of D**
2.000	(0,0)	
-3.000	(2,3)	
0.000	(-1,4)	
1.250	(2,-5)	
Undefined	(3,1)	
-2.667	(-1,-2)	
1.125	(-3,2)	
3.500	(4,0)	

Activity 2: Slope and Lattice Points

- *Objectives*
    - *Use a slider to explore the slope of a line and its lattice point.*
    - *Explore the file to answer the questions in the Exeter Lab.*

Preparing the Window

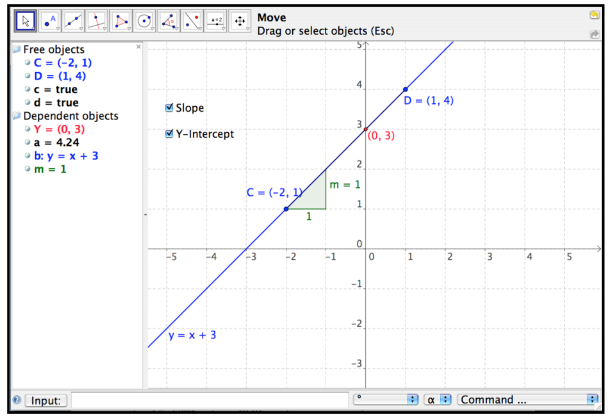
- Open a new GeoGebra file.
- Keep the algebra window open and the axes on. Turn the grid on. Set labeling to new points only.
- Set Point Capture to **on (grid)**

Construction Steps

1		Create a check box and change the caption to Slope (you will link things to the text box later)
2		Create a line with through two points (rename the points C and D and show the lable as name and value)
3		Create the slope by clicking the line created in step 2
4	ABC	Create a text box and type "=" + m (the red text will be static text)
5		Link the slope from step 3 and the text box from step 4 by highlighting them in the properties window and in the advanced tab type the name of the Boolean value (check box).
6		Create a check box and change the caption to Y-Intercept
7		Create the intersection point of the line from step 2 and the y axis.

Format the file as you like lock down the text box and the sliders. Next save the file as **Linear42.ggb**. Use this file to complete the following Exeter Lab

Exeter Computer Lab #2 – Slope and Lattice Points



1. Once you have launched GeoGebra, click on menu **File/Open**, and choose file entitled ***Linear42.ggb***. When the file opens, you should see a coordinate grid with a line constructed through the two points labeled C and D. The coordinates of C and D should be displayed next to the points. Click the check boxes entitled "Slope" and "*Y*-Intercept." The equation of the line will be displayed in slope-intercept form, $y = mx + b$, near the edge of the window.

2. Points C and D completely determine the line. The program is in a mode that only allows points C and D to be dragged to locations with integer coordinates. However, the *y*-intercept may end up at a non-integer location, but not through dragging on it. Only points C and D are to be moved in the exercises that follow. Drag point C to the origin, and then move D to the appropriate location so as to produce the line whose equations are given below. Record the location of point D for each of these lines.

Equation of Line	Coordinates of C	Coordinates of D
$y = 4.000x$	(0,0)	
$y = -3.000x$	(0,0)	
$y = 1.500x$	(0,0)	
$y = 0.000$	(0,0)	
$y = -1.400x$	(0,0)	
$x = 0.000$	(0,0)	
$y = 2.750x$	(0,0)	
$y = -0.\overline{33}x$	(0,0)	
$y = 0.375x$	(0,0)	

3. Complete the chart below by filling in locations for both C and D so as to produce the line whose equation is given in the first column of the table.

Equation of Line	Coordinates of C	Coordinates of D
$y = 2.000x - 3.000$		
$y = -1.500x + 4.000$		
$y = 2.000$		
$y = 1.750x - 1.000$		
$x = 4.000$		
$y = -1.600x + 1.000$		
$y = 1.333x - 5.000$		

4. This exercise is exactly like #3 above, except the y-intercepts are not at lattice points. This only means that neither C nor D will be located on the *y*-axis. Find locations for C and D that will form the line given in the left column of the table below.

Equation of Line	Coordinates of C	Coordinates of D
$y = 1.500x - 2.500$		
$y = -0.600x + 1.800$		
$y = 2.750x + 0.500$		
$y = 1.333x - 2.667$		
$y = -0.375x + 1.250$		

5. Explain in the space below why there are an infinite number of lattice points on any one of the lines given above.

6. In the space below, explain or demonstrate why there are no lattice points on the line:

$$y = 1.2x + 1.5$$

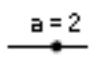
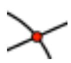
Activity 3: Slope-Intercept Sliders

- *Objectives*
  - *Use a slider to explore the slope and y-intercept of a line.*
  - *Explore the file to answer the questions in the Exeter Lab.*

Preparing the Window

- Open a new GeoGebra file.
- Keep the algebra window open and the axes on. Turn the grid on. Set labeling to new points only.

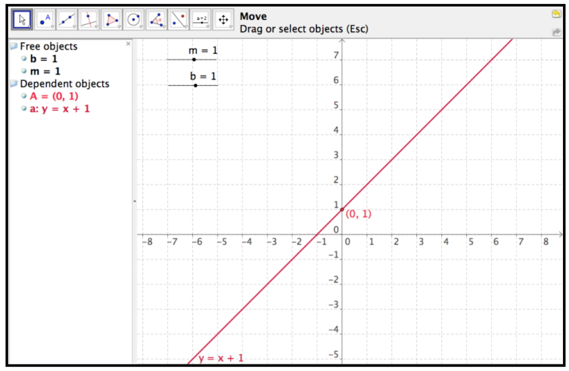
Construction Steps

1		Create sliders m and b. Set the minimum at -10 and the maximum at 10. Set the increment to 0.1.
2		Into the input field, enter the formula: f(x) = m*x+b
3		Create the intersection of the line from step 2 and the y axis (show the label as value)

Format the file as you like lock down the text box and the sliders. Next save the file as **Linear43.ggb.** Use this file to complete the following Exeter Lab

Exeter Computer Lab #3 – Slope-Intercept Sliders

Once you have opened GeoGebra, click on the menu **File/Open**, and choose the file entitled ***Linear43.ggb***. When the file on the screen opens up, you should see a sketch like the one on the right. There are two "*sliders*" that you will use to change the value of the *parameters m* and *b* in the equation $y = mx + b$. These two parameters control the line in red that is displayed on the screen. Also displayed are the values for *m*, *b*, and the *y*-intercept of the red line.



General Instructions: The initial values for *m* and *b* are initially set to 1.0. For the purpose of this worksheet, the accuracy for these parameters has been purposefully chosen to be one decimal place. It is important for you to realize this limitation on the accuracy, so that for all problems you should check your answers by hand calculations in order to make any necessary small adjustments. Technology is a marvelous tool, but always know the limitations of the technology you are working with. You will be dragging on the points along the sliders that are labeled *m* and *b*. This will change the values of these parameters and thus change the red line in the sketch.

1. Click and drag on the slider labeled *m*. Describe below what happens to the red line when *m* becomes increasingly positive.

2. What happens to the line if *m* equals zero?

3. Describe below what happens to the red line when *m* becomes increasingly negative.

4. Summarize the role *m* plays in the equation $y = mx + b$.

5. Drag the slider *m* so that its value is back to 1.0. Now drag slider point *b* and observe. Describe below what happens to the red line as *b* changes. Focus your attention on the *y*-intercept.

6. Drag slider point *b* so that its value is 3.0. Now move slider point *m* back and forth. Move the slider point *b* so that its value is -2.0. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

7. Summarize the role *b* plays in the equation $y = mx + b$.

8. Write the equation in slope-intercept form for each of the lines described below. As a check, drag the *m* and *b* slider points to the appropriate values so that the line described is drawn on the screen. Remember, the accuracy of the slider values is only to one decimal place. On some problems you will need to do some paper and pencil calculations as a check to make sure you have the correct equation of the line.

a) Slope is 2.0, and the y-intercept is (0, -3).

b) Slope is -1.5 and the y-intercept is (0, 4).

c) Slope is 3.0 and the *x*-intercept is (-2, 0).

d) Slope is -0.4 and contains the point (-2, 4).

e) Contains the points (3, 5) and (-1, 3).

Activity 4: Point-Slope Form

- *Objectives*
  - *Use a slider to explore the affects of certain parameters on the point-slope form of a line.*
  - *Explore the file to answer the questions in the Exeter Lab.*

Preparing the Window

- Open a new GeoGebra file.
- Keep the algebra window open and the axes on. Turn the grid on. Set labeling to new points only.

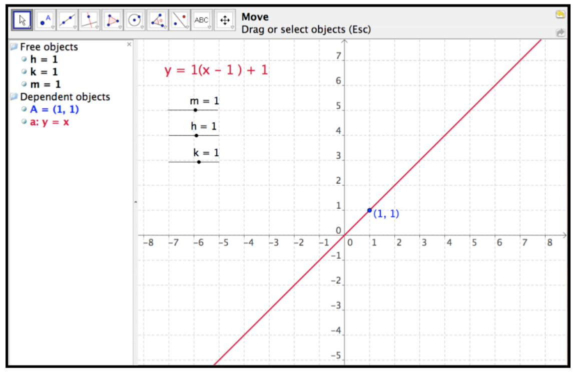
Construction Steps

1	a = 2	Create sliders m, h, and k. Set the minimum at -10 and the maximum at 10. Set the increment to 0.1.
2		Into the input field, enter the formula: f(x) = m*(x-h)+k
3	ABC	Create a text box and type "f(x)="+m+"(x-"+h+")+"+k (the red text will be static text)
5		Into the input field, enter the formula: (h,k).

Format the file as you like lock down the text box and the sliders. Next save the file as **Linear44.ggb.** Use this file to complete the following Exeter Lab

Exeter Computer Lab – Point-Slope Form Sliders

1. Once you have opened the program GeoGebra, click on menu **File/Open**, and choose file entitled **Linear44**. When the file on the screen opens up, you should see the one on the right. There is a coordinate axis with three "*sliders*" you will use to change the value of the parameters *m*, *h*, and *k*. These three parameters control the line in red that is displayed on the screen. Also displayed in the upper left hand portion of the screen is the equation in point-slope form.



General Instructions: The initial values for *m*, *h*, and *k* are all initially set to 1.0. For the purpose of this worksheet, the accuracy for these parameters has been purposefully chosen to be one decimal place. It is important for you to realize this limitation on the accuracy, so that for certain problems you will check your answers by hand calculations in order to make any necessary small adjustments. Technology is a marvelous tool, but always know the limitations of the technology you are working with. You will be dragging on the points along the sliders that are labeled *m*, *h*, and *k*. This will change the values of these parameters and thus change the red line in the sketch whose equation is determined by these parameters in the form $y = m(x - h) + k$.

2. Click and drag on the slider labeled *m*.

a) Describe below what happens to the red line when *m* becomes increasingly positive.

b) What happens to the line if *m* equals zero?

c) Describe below what happens to the red line when *m* becomes increasingly negative.

d) Summarize the role m plays in the equation $y = m(x-h)+k$.

3. Drag slider point *m* so that it's value is equal to 1.0. Now drag slider point *h* and observe.

a) Describe below what happens to the red line as *h* changes. Focus your attention on the point (*h, k*) that is on the line.

b) Drag slider point *h* so that its value is 3. Now move slider point *m* back and forth. Move slider point *h* so that its value is -2. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

c) Summarize the role *h* plays in the equation $y = m(x-h)+k$.

4. Drag slider points *m* and *h* so their values are one. Now drag slider point *k* and observe.

a) Describe below what happens to the red line as *k* changes. Focus your attention on the point (*h,k*) that is on the line.

b) Drag slider point *k* so that its value is 3. Now move slider point *m* back and forth. Move slider point *k* so that its value is -2. Move slider point *m* back and forth again. Describe below what behavior in the red line you observe.

c) Summarize the role *k* plays in the equation $y = m(x-h)+k$.

5. Write the equation of the line in *point-slope form* for each of the lines described below. As a check, drag the *m*, *h*, and *k* slider points to the appropriate values so that the line described is drawn on the screen. Remember, the accuracy of the slider values is only to one decimal place. On some problems you will need to do some paper and pencil calculations as a check to make sure you have the correct equation of the line.

a) Slope is 2 and contains the point (1,3).

b) Slope is 2 and contains the point (-2,1).

c) Slope is -1 and contains the point (-2,1).

d) Parallel to the line $y = 3(x - 2) + 4$, and goes through the origin.

e) Slope is 4/5 and the x-intercept is (2, 0).

f) Contains the two points (2,3) and (-1,4).

g) Contains the two points (-3,5) and (4,5)

6. Explain below why the line through the points (2, 3) and (2, -2) cannot be constructed using this slider program, implying that its equation cannot be written in *point-slope form.*

<u>Closing (30 min):</u>

Analyzing Escher Tessellations

Presenter Information

- *Objectives*
  - *Participants will analyze non-polygonal tessellations created by M.C. Escher and discover the underlying polygonal tessellation from which they were created.*
  - *Participants will discover the relationship between the transformations used to create a tile and the symmetries found in the tessellated plane.*

Introduction

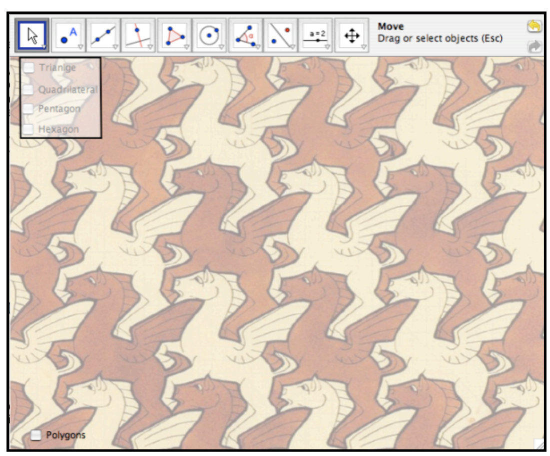
M.C. Escher was the master of transforming polygonal tessellations into non-polygonal works of art. The connection between the symmetries of the underlying polygonal tessellations used by Escher, and the symmetries of his transformed tessellations can be traced back to the creation of his non-polygonal tiles. In this section, we will be using three GeoGebra files “pegasus.ggb”, “dogs. ggb”, and “lizards. ggb” to explore these connections.

Pegasus

Open the GeoGebra file, *pegasus.ggb*. This file is intended to help participants recognize and generalize a rule for finding the underlying polygonal tessellations that M.C. Escher used to create his works of art. An image of Escher’s *Pegasus* was imported into GeoGebra and has been locked to the background.

Give the participants a minute to examine and describe the features of the tessellation while you pose the following questions:

1. What polygon was used to create this tessellation?
2. Where are the vertices of these underlying polygons located?
3. What transformations could be used to tile the plane using one Pegasus?
4. What type of symmetry does this tessellation have?

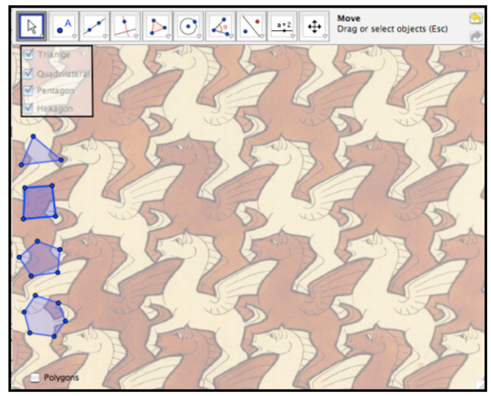


5. After giving them a minute or two to think about these questions, spend about two minutes going over each one.

1. What polygon was used to create this tessellation?

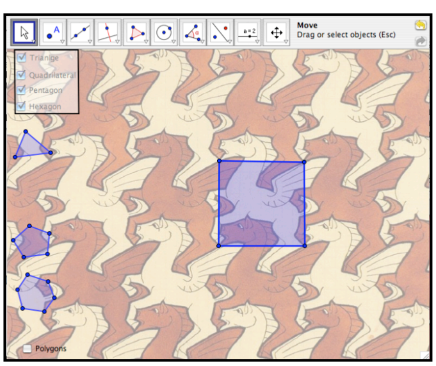
Click the check box for the triangle, quadrilateral, pentagon, and hexagon. Ask the participants which of the following polygons was used to create this tessellation.

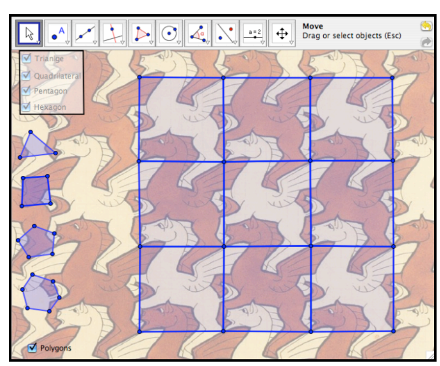
Entertain suggestions by dragging some polygons out into the drawing pad. Click in the middle of the polygon to do this. Then drag the vertices to give the polygon special features like parallel sides, congruent angles, etc. To transition into the next question, we might need to lead the participants to the correct answer. A square was used to create this tessellation. Some participants should recognize this, but often have a difficult time describing how they came to this conclusion. Parallel lines can be visualized if we move our eyes across the rows formed by the tops of the heads. Likewise, each Pegasus seems to stack on top of another one forming columns.



Pull the quadrilateral into the shape of the square and ask the participants where the vertices should be placed.

2. Where are the vertices of these underlying polygons located?
Have the participants visualize a tessellation of squares only. Ask them how many squares come together at each vertex. They should be able to see four. Have them now find the locations where four of the Pegasus tiles meet. Use the point tool to mark some of these locations.

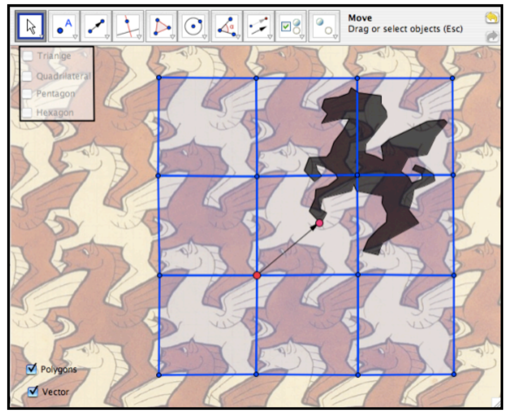




Select the “Polygon” check box at the bottom of the screen to bring forth a part of the underlying tessellation.

3. What transformations could be used to tile the plane using one Pegasus?

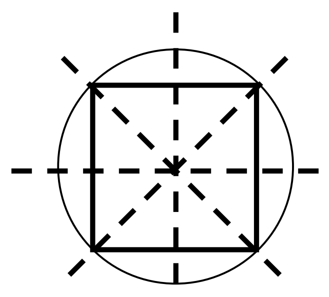
Could we use reflections, rotations, translations, or some combination of these transformations?



Every Pegasus is oriented in the same leftward facing direction. In addition, they could be viewed as stacked on top of each other. Through the use of color, they can be visualized moving in a diagonal fashion as well. All of these moves could be accomplished with translations.

Select the "Vector" check box. A red dot will appear at a vertex in the underlying grid. Drag this dot and the Pegasus will move, for it is the head of a vector that translates the black Pegasus. Have the participants describe the translations that allow the Pegasus tile to fit into place.

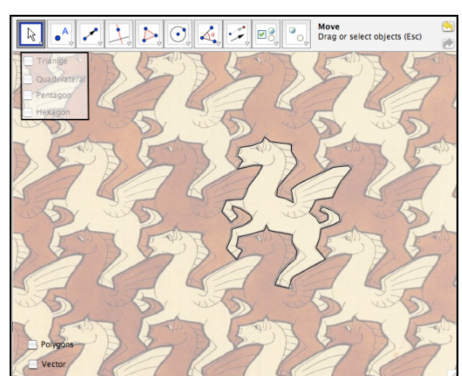
4. **What type of symmetry does this tessellation have**?



What is a symmetry transformation? It is a transformation that leaves an object unchanged: "Something that does nothing." Rotating a square, 90 degrees about its center, is a symmetry transformation. Have the participants visualize a square. Ask them: What types of transformations could be done to a square such that it would appear as if nothing had been done?

In the same respect, what transformation could be done to the entire tessellated plane such that it would appear as if nothing had changed?

Have the participants visualize picking up the entire plane, doing something to it, and putting it back down such that everything looks the same.

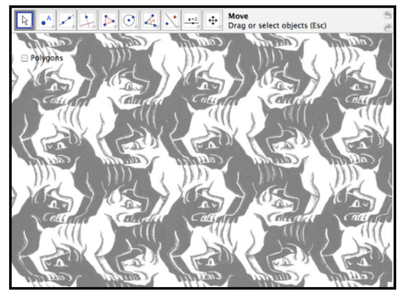


Participants should state that the tessellation has translational symmetry. The translations from question 3 would cause the transformed plane to coincide with itself.

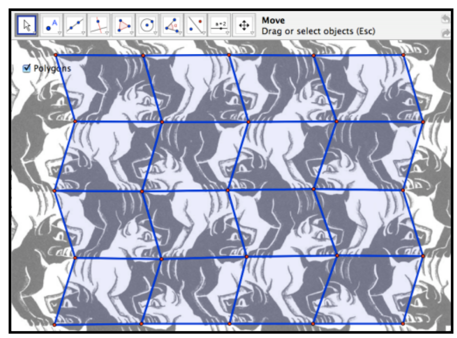
Dogs

Open the GeoGebra file, *dogs.ggb* to provide participants with a quick example to practice what they just learned. The following questions should be asked to the participants:

1. What polygon was used to create this tessellation?
2. What type of symmetry does this tessellation have?



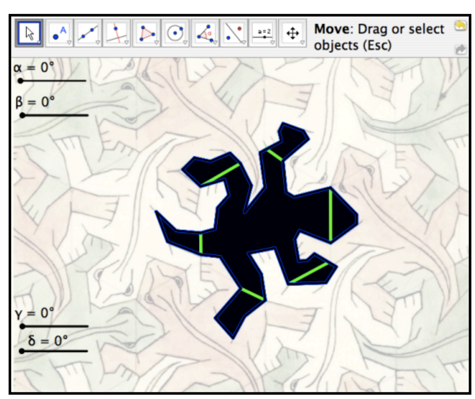
Use the point to place points at locations where ***more than two*** dogs meet. This is the general rule for finding the underlying lattice of polygons in Escher's non-polygonal tessellations. Why is it ***more than two***? Can we have a tessellation where there are two or less polygons coming together at a vertex?



Select the "Polygon" check box. Notice that the parallelograms seem to changing direction in every row. As a result, the dogs are facing opposite directions. Ask the participants what transformation(s) would be needed to get from dog to dog. Translations and glide reflections will do the trick. These will also question number 2 above.

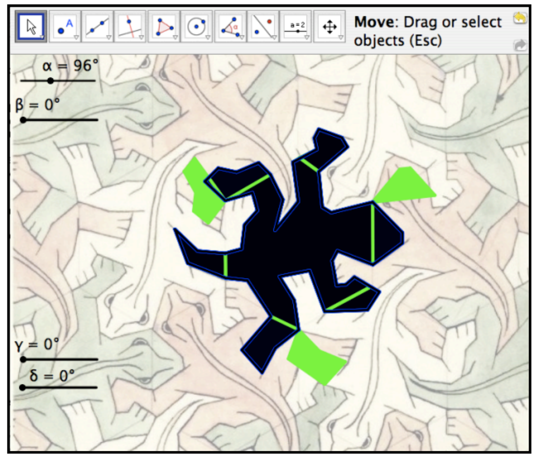
Lizards

Thus far, the participants have learned how to discover the underlying lattice of an Escher tessellation while discussing the symmetries inherent in each work of art. Participants have not discussed how the tiles are formed from their underlying polygon. While exploring the Lizards, participants will link the tile formation process to the symmetries of the tessellation.



Open the GeoGebra file, *lizards.ggb*. Without moving any sliders, allow the participants to state what the underlying polygonal tessellation is made from and what type of symmetries it has. They should respond that the tessellation is made from hexagons and has rotational and translational symmetries. If points are placed where any **three** lizards meet, the hexagons will emerge. The rotational symmetries can be seen at locations where the heads are next to each other. This can be found in other places as well. The lizards traveling in the same direction that are the same color will help the participants visualize the translational symmetry. To help the participants see the above symmetry transformations, move only sliders Alpha and Beta to their maximums and back again a few times to give insight into the tile formation process. While moving the sliders, ask and discuss the following questions:

1. Describe how the tile was formed from its underlying polygon? Use precise vocabulary.
2. Identify the vertices where the tile building transformations are taking place. Describe their location on the underlying polygon.
3. What transformation(s) could be used to move the tile into the spots of other lizards?



Make sure the following points are brought out while discussing the above questions:

1. **Describe how the tile was formed from its underlying polygon? Use precise vocabulary**.

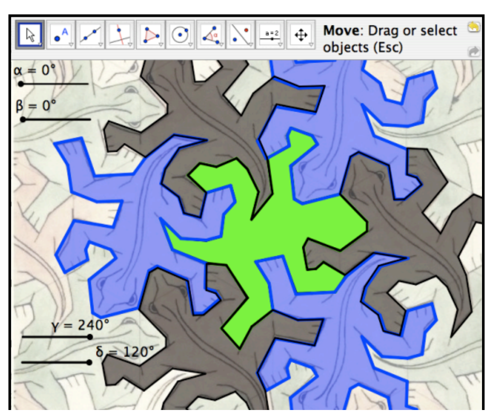
After moving sliders Alpha (α) and Beta (β), the participants should see that pieces were cut out of the hexagon. They were then rotated and attached back adjacent to remaining pieces of the hexagon's edges.

2. **Identify the vertices where the tile building transformations are taking place. Describe their location on the underlying polygon**.

All of the rotations are taking place around three alternating vertices in the hexagon. Make sure that the participants visually locate these three vertices. The pieces that move with slider Alpha (α) are easier to track than the ones which move with slider Beta (β).

3. **What transformation(s) could be used to move the tile into the spots of other lizards?**

Take input from the participants. Then, one at a time, move the sliders gamma (γ) and delta (δ) at the bottom of the page. Describe the transformations taking place using precise vocabulary.



In addition to the translational symmetry implied by the use of color, do the transformations above describe the symmetries of the tessellation?

Use the Alpha (α) slider to discuss how the movements of the blue lizards relate to creation of the original tile. The blue lizards will fit into the holes created by the Alpha (α) slider. Participants should see they follow the same path.

Use the Beta (β) slider to discuss how the movements of the black lizards relate to creation of the original tile. As with the tiles moved by the Alpha (α) slider, they tiles moved with the Beta (β) slider provide the holes for the black lizards to fit into.

Conclusion

In the activities above, participants discovered how to extract the underlying polygonal tessellation from which Escher created his works. In comparing the non-polygonal tile to its underlying polygon, keeping in mind the transformations used to create each tile, participants were able to link the symmetries of the tessellation to the tile bearing transformations. If time permits, return to Pegasus and Dogs to discuss how each of those tiles is created.