## Practice Block 1

## Activity1: Explore reflections about lines and points, translations, and rotations

## Reflections (Mirror Lines)

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Reflection of a polygon with respect to a line:

| 1 | Construct Triangle ABC |  |
| :--- | :--- | :--- |
| 2 | - | Construct Line DE |
| 3 | $\bullet$ | Reflect Triangle ABC with respect to Line DE |

## Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B ' $\mathrm{C}^{\prime}$ ?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point $A$.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in opposite order. For example, the reflection of Quad $A B C D$ could be named Quad D' ${ }^{\prime} B^{\prime} A$ '.

Can't move Triangle $A^{\prime} B^{\prime} C^{\prime}$ because it is just the image of Triangle $A B C$. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is double the distance from any of the points to the line of reflection. In other words the line of reflection would be the perpendicular bisector of the segment joining a point and its image.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of reflecting an object with respect to a line is an isometry. A transformation that preserves distance and angle measure is an isometry.

Add a Free Point G to the worksheet. Without using the reflection tool, how would you locate point G' (reflected about Line DE)?

- Reflected Point Construction One
- Reflected Point Construction Two

If you suspected that two figures were congruent and related by a reflection, how could you confirm your guess and reconstruct the reflection line?

## Reflections (Mirror Point)

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Reflection of a polygon with respect to a point:

| 1 | - | Construct Triangle ABC |
| :--- | :---: | :--- |
| 2 | $\bullet$ A | Construct Point D |
| 3 | $\bullet \bullet$ | Reflect Triangle ABC with respect to Point D |

## Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- Is there another transformation that is equivalent to this one?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point $A$.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad $A B C D$ could be named Quad A 'B'C'D'.

Can't move Triangle $A^{\prime} B^{\prime} C^{\prime}$ because it is just the image of Triangle $A B C$. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The other transformation that would be the same is a 180 degree rotation (clockwise or counter-clockwise) with respect to the same point.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of reflecting an object with respect to a point is an isometry. A transformation that preserves distance and angle measure is an isometry.

## Translations

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Translation of a polygon with respect to a vector:

| 1 | Construct Triangle ABC |  |
| :--- | :--- | :--- |
| 2 | Construct Vector DE |  |
| 3 |  | Translate Triangle ABC using Vector DE |

## Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point $A$.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad $A B C D$ could be named Quad A 'B' $C^{\prime} D^{\prime}$.

Can't move Triangle $A^{\prime} B^{\prime} C^{\prime}$ because it is just the image of Triangle $A B C$. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is equal to the magnitude of the vector. In addition, the angle at which the pre-image has been slid to the same as the relative angle of the vector..

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of translating an object with respect to a vector is an isometry. A transformation that preserves distance and angle measure is an isometry.

## Rotations

Participants should open a new GeoGebra window. Hide the algebra window, axes, and grid (if necessary).

Rotation of a polygon with respect to a point:

| 1 | C | Construct Triangle ABC |
| :---: | :---: | :--- |
| 2 | A | Construct Point D |
| 3 | $\stackrel{a}{0}=2$ | Make angle slider |
| 4 | - | Rotate Triangle ABC about point D using the angle slider |

## Discussion questions:

- How do the labels of the points on the image object compare to those on the pre-image object?
- Why can't you move Triangle A'B'C'?
- What is the distance between a point on the pre-image and its reflected point on the image?
- How do the two triangles compare?

Labels of reflected points use the same letter with the exception of the mark in the upper right hand corner, ex. A' (read: A prime). This is to denote that it is the reflected image of point $A$.

If you name the Triangle (polygon) in counter clockwise order then the naming of the image could be read in the same order. For example, the reflection of Quad $A B C D$ could be named Quad A 'B' $C^{\prime} D^{\prime}$.

Can't move Triangle $A^{\prime} B^{\prime} C^{\prime}$ because it is just the image of Triangle $A B C$. The location of image is dependent on the location of the pre-image. Show the Algebra Window and notice the Free and dependent objects.

The distance between the point on the image and the pre-image is a complicated issue in this case. Some triangle trigonometry may have to be used. Unless distance is meant to be circular distance using the radius and angle of rotation.

The two triangles (polygons) corresponding sides and angles are congruent. The two triangles (polygons) are congruent. We can say: In a plane the process of translating an object with respect to a vector is an isometry. A transformation that preserves distance and angle measure is an isometry.

## Activity 2: Explore reflecting over parallel and intersecting lines Reflecting over parallel lines

- Overview
- Discuss how multiple reflections affect an object
- Guided construction polygon reflected over two parallel lines.


## Discussion:

- How would reflecting over two parallel lines affect an object?
- Could there be other transformations that are isomorphic to reflecting over two parallel lines? If so, find one.


## Guided construction polygon reflected over two parallel lines.

- Preparations
- Open new GeoGebra file
- Hide algebra window
- HIde coordinate axes
- Hide grid (if necessary)
- Construction steps

| 1 |  | Construct Line AB |
| :--- | :--- | :--- |
| 2 | A | Construct Point C, not on Line AB |
| 3 | - | Construct a line parallel to Line AB thru Point C |
| 4 | Construct polygon (exterior of parallel lines) |  |
| 5 | - | Reflect polygon with respect to Line AB |
| 6 | - | Reflect reflected polygon with respect to line thru Point C |

- Check construction
- Drag the original polygon (Which is the original polygon? How do you know which is the original polygon?)
- Discussion

1. How does moving the original polygon affect the first reflected polygon?
2. How does moving the original polygon affect the second reflected polygon?
a) Is there a single transformation that can be done on the original polygon to create the second reflected polygon?
b) If you answer to part a) is yes, then create the transformation in GeoGebra.
c) Can you argue why there is/isn't a single transformation that could replace the double reflection above?

## Reflecting over two intersecting lines

- Overview
- Discuss how multiple reflections affect an object
- Guided construction polygon reflected over two intersecting lines.


## Discussion:

- How would reflecting over two intersecting lines affect an object?
- Could there be other transformations that are isomorphic to reflecting over two intersecting lines? If so, find one.


## Guided construction polygon reflected over two parallel lines.

- Preparations
- Open new GeoGebra file
- Hide algebra window
- HIde coordinate axes
- Hide grid (if necessary)
- Construction steps

| 1 |  | Construct Line AB |
| :--- | :--- | :--- |
| 2 |  | Construct Line $A C$ |
| 3 |  | Construct polygon (exterior of parallel lines) |
| 4 | - | Reflect polygon with respect to Line AB |
| 5 | - | Reflect reflected polygon with respect to Line AC |

- Check construction
- Drag the original polygon (Which is the original polygon? How do you know which is the original polygon?)
- Discussion

1. How does moving the original polygon affect the first reflected polygon?
2. How does moving the original polygon affect the second reflected polygon?
a) Is there a single transformation that can be done on the original polygon to create the second reflected polygon?
b) If you answer to part a) is yes, then create the transformation in GeoGebra.
c) Can you argue why there is/isn't a single transformation that could replace the double reflection above?
