Triangle Inequality Theorem

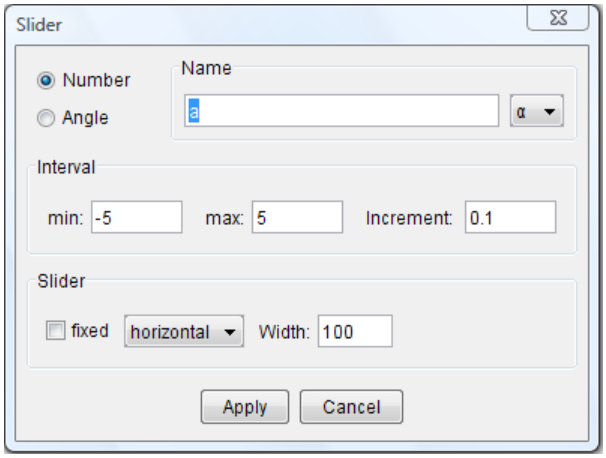
Preparation

- Open a new GeoGebra file
- For this construction, we will not use the coordinate axes or the Algebra window. Click on the *View* menu on the top of the page. Select *Axes* and click on it to inactivate it. Open the *View* menu again, select *Algebra* window and click to hide it or click on the *x* located on the upper right hand side of the algebra window.

Construction process

We are going to construct a triangle with given side lengths and later be able to change the side lengths. For that we are going to create sliders.

- Select the slider tool  and click on the drawing pad. A new window will appear.

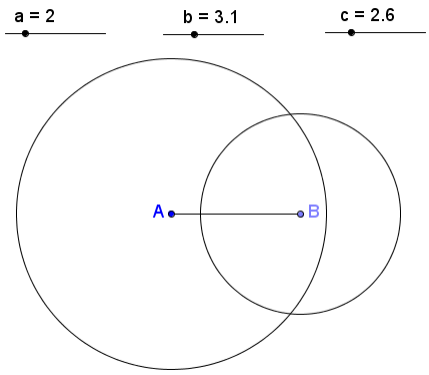


- The slider will represent the length of one side of the triangle. The name of the Number (slider) is by default *a*. The default settings suggest a minimum value of -5 and maximum value of 5. We are going to change the minimum value to 0 and the maximum to 10. Leave the increment as it is. Click the button *Apply*.
- We need 2 more sliders for the other two sides of the triangle. Repeat the process twice. The names of the new numbers will be, by default, *b* and *c*.
- Select the move tool  and drag the point on the sliders. Notice the numbers changing from 0 to 10 in increments of 0.1.
- Select the tool *Segment with given length from point,* found under the *Line* toolbox  . Click anywhere in the drawing pad to create a point. A window will appear asking for the length. Type the letter *c* (name of the slider). The segment

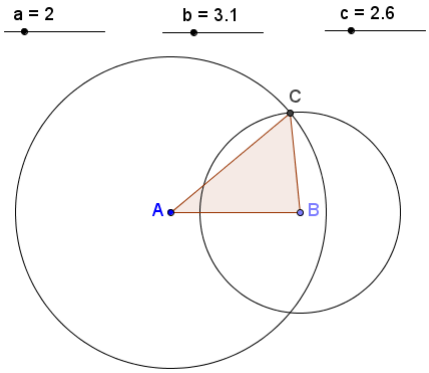
will be created. Select the move tool and change slider c. Notice how the segment increases and decreases in length as the value in the slider changes.

- Right-click (MacOS: Ctrl-click) on one of the endpoints of the segment, select *show label*. Do the same for the other end point.

- In order to construct the other two sides of the triangle, we will use the *Circle with center and radius* tool  , located in the circle toolbox. Click on point A (center of the circle). A window will appear asking for the radius, type letter *b*. A circle will be constructed that has radius *b*. Select the *Move* tool and change slider *b*. Notice how the radius of the circle changes.
- Activate the *Circle with center and radius* tool again. Click on point B (center of the circle and this time type letter *a* for the radius of this last circle. Move the sliders, verify your circles change as the values of the sliders *a* and *b* change.



- If your two circles, do not intersect, move slider *a* or *b* until they intersect. Under the *point* toolbox  select *Intersect two objects*  . Move the pointer close to the intersection, notice the two circles getting darker, click on the intersection to create the point.
- Right-click (MacOS: Ctrl-click) on the intersection point and select *Show label*. This will be point C.
- Using the Polygon tool  click on the points A, B, C, and A in order to create the vertices of a polygon. Connect the last and first vertex to close the polygon. Always connect vertices counterclockwise! (this is necessary for measuring the inside angles of the polygon)



- Select the move tool and move the points on the sliders. Notice how the triangle changes. Do we always have a triangle? Make a conjecture about your observations, what length should the sides be so we can have a triangle?

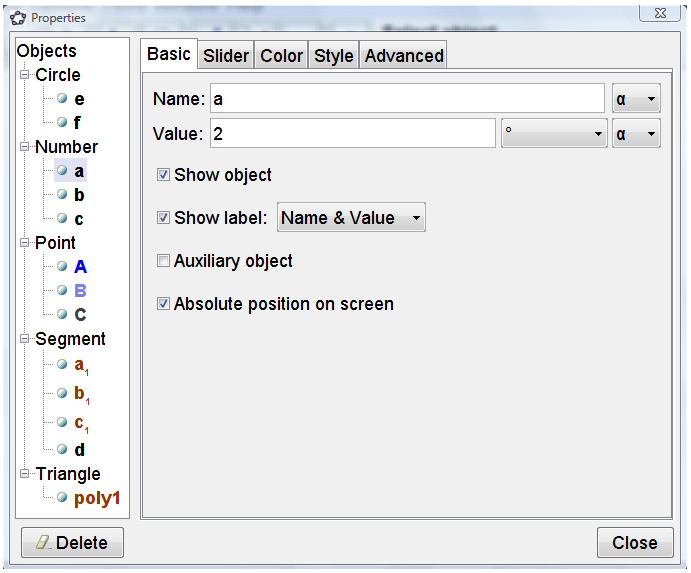
Use the Properties dialog to enhance your construction

There are different ways to access the Properties dialog:

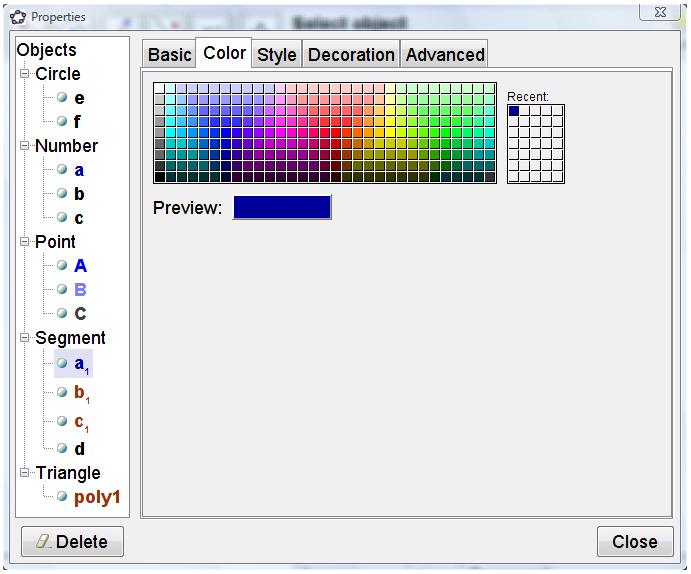
- Right-click (MacOS: Ctrl-click) on an object. Select *Properties…* from the Edit menu
- Double click an object in *Move* mode

We will change the colors of the sliders, the sides of the triangles, and the circles so we can visually tell what slider changes what side.

- Right-click (MacOS: Ctrl-click) or double click on slider *a* and select *Properties*. A window will appear

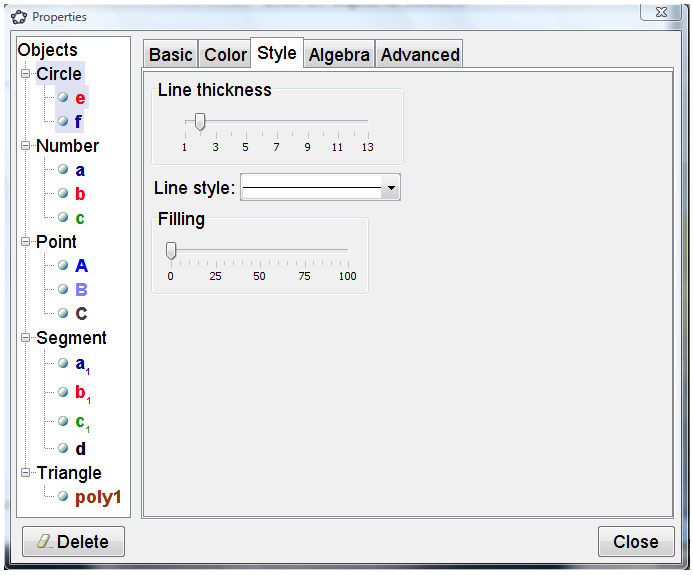


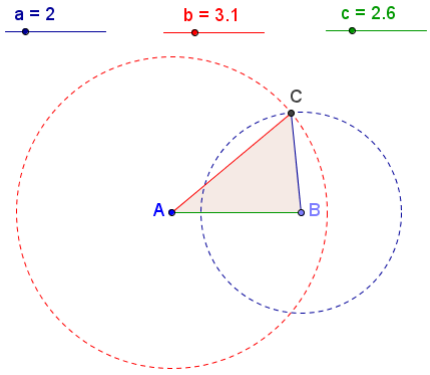
- Select the tab Color and pick any color you like



- From the list on the left side of the Properties window, find Segment: a_1. Select segment a_1 by clicking on the name a_1 and use the same color you selected for number *a*. Select circle f and use the same color.
- On the left list, select number *b* and pick a different color. Use the same color for segment b_1.
- Finally, select number *c* and a third color. Use the same color for segment c_1 and circle *e*.

- On the left, click on the work *Circle* to select both circles at the same time, under the tab *Style* you will find *Line style*, click on the down arrow and select a dashed style for the circles.





- Close the Property window.

Measuring the length of the sides of the triangle

On the *Measurement* tools menu,  , find the *Distance or length tool*,  . This tool could be used in two different ways:

a. To measure distance between 2 points
b. To measure length of a segment

Use the tool, either by clicking on 2 endpoints at a time or by clicking on a segment. Find the length of the 3 sides of the triangle; verify that is the same as the values on the sliders.

Inserting text into the Drawing pad

Text can be static and dynamic.

Inserting static text

Activate the  *Text* tool and click anywhere on the drawing pad.

- Type the following text into the appearing window:
  "*Triangle Inequality Theorem:*
  The sum of the lengths of any two sides of a triangle is greater than the length of the third side."
- Click *Apply*.
- Adjust the position of the text using the *Move* tool.

Hints: You can change the properties of the text in the *Properties dialog* (e.g. edit text, font style, font size, formatting). On tab *Basic* you can fix the position of the text so it can't be moved accidentally any more.

Inserting dynamic text

Dynamic text refers to existing objects and adapts automatically to modifications, for example the length of the sides as the sliders move. In our example, we want to show that the triangle will exist when the following conditions are true: $a + b \geq c$, $a + c \geq b$, and $b + c \geq a$

- Activate the  *Text* tool and click on the drawing pad.
- Type $a + b \geq c$ into the appearing window, and click *Apply*.

  Hints: to find the $\geq$ symbol, open the window  located on the right hand side of the text dialog box.
  This is static text and won't change if the sliders are moved.

- With the *text* tool still activated, click on the drawing pad again. Insert dynamic text by clicking on segment *a* in the graphics window.
  - GeoGebra will insert the name of the segment into the text field
- To add the + sign, type the following: (do not leave an empty space)+" + ". Only leave spaces inside the quotation marks.
  - The first + sign connects the dynamic and static part of the text
  - Quotation marks around any text indicate it will be static.
- Click on segment *b.*
  - Additionally, GeoGebra adds a + symbol to connect the new dynamic part to the rest of the text.
- Add the = sign by typing +" = "
- Click on segment *c.*
- Click *Apply*.
- Observe how the dynamic text changes as you move the sliders

Task:

Insert the static and dynamic text for $a + c \geq b$ and $b + c \geq a$.

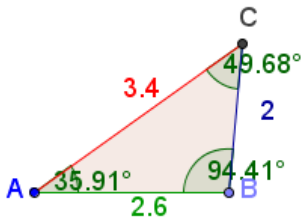
Hide/Show Objects

The circles were necessary for the construction of the triangle but, for the following investigations, are not necessary. Activate the *Hide/Show Objects* tool  and click of the circles. You will still see them while the tool is active. Activate the *Move* tool, the circles will disappear. If you delete the circles, the triangle will disappear because its construction is dependant on those circles.

Interior Angles

We can find the measures of the interior angles of the triangle by using the *Angle* tool  .

- Activate the tool
- Click one time inside of the triangle. GeoGebra will give you the measure of the three interior angles at once.



- Is there any relationship between the measure of the angle and the length of the opposite side? Make a conjecture. Move the sliders to verify your conjecture.

Area of the triangle and Altitudes

- Hide the interior angles
- Activate the *Area* tool  and click inside of the triangle
- You can move the text by activating the *Move* tool and dragging the text.

If we assume side AB to be the base of the triangle and knowing the area, what is the altitude?

- Using the *Line trough two points* tool  draw line AB by clicking on point A and then point B.

- The altitude is perpendicular to side AB and passes trough point C. Activate the *Perpendicular line* tool  . Click on point C and then on line AB (you can read the instruction in the help menu)
- Using the *Intersect two objects* tool, make a point at the intersection of the perpendicular line and line AB. You do not need to be at the intersection point, click anywhere on the perpendicular line and then anywhere on the line AB.
- Hide the perpendicular line and line AB
- Create a segment using the *Segment between two points* tool  click on point C and the newly created point of intersection.
- Right-click (MacOS: Ctrl-click) on the segment and using the *Properties…* dialog, change the color, thickness or style of the segment.
- Show the right angle, use the Angle tool and click on three points counterclockwise: B, intersection, C. Right-click on the angle and click on *Show label.* This will actually hide the label since it was showing before.
- Move slider *a*. Why does the right angle change? GeoGebra measures the angles counterclockwise, if the point of intersection goes to the right of point B, Geogebra gives you the measure of 270°.
- Find the length of the new segment. Is the length what you had calculated before? If you move slider *a* only, what is the highest value the altitude can take?
- As you move slider a, point C moves. What is the path that point C describes? Right-click on point C, select *Trace on.* Move slider *a*, is the path what you had predicted?

Saving GeoGebra files

You might want to create a new folder called GeoGebra_Workshop on your desktop to store all your work.

Save your drawing

- Open the *File* menu and select *Save*.
- Select the folder *GeoGebra_Workshop* in the appearing dialog window.
- Type in a *name* for your GeoGebra file.
- Click *Save* in order to finish this process.

Hint: A file with the extension '*.ggb*' is created. This extension identifies GeoGebra files and indicates that they can only be opened with GeoGebra.

Hint: Name your files properly: Avoid using spaces or special symbols in a file name since they can cause unnecessary problems when transferred to other computers.

Instead you can use underscores or upper case letters within the file name (e.g. Triangle_Inequality.ggb).

What to practice

- How to open a new GeoGebra window (menu *File – New window*).
- How to open a blank GeoGebra interface within the same window (menu *File – New*)
  Hint: If you didn't save the existing construction yet GeoGebra will ask you to do so before the blank screen is opened.
- How to open an already existing GeoGebra file (menu *File – Open*).
  - Navigate through the folder structure in the appearing window.
  - Select a GeoGebra file (extension '*.ggb*').
  - Click *Open*.